



ENGINEERING TRAINING
DEPARTMENT

DC TRANSIENTS :- the pulse response of CR and LR circuits

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LIST OF EQUIPMENT REQUIRED FOR PRACTICAL WORK

Oscilloscope

DC power supply

AF signal generator

AVO

Breadboard plus leads etc.

Resistors

Capacitors

Inductors

1k

10nF

100mH

4.7k

100nF

10k

1000 μ F

47k

DC TRANSIENTS

1. INTRODUCTION

- 1.1 This package is concerned with the effect of changing the applied voltage or the current flowing through series CR or LR circuits in a non sinusoidal manner.

In particular the response of such circuits to repetitive voltage and current pulse waveforms is considered in detail as these circuits are often found in broadcasting equipment.

For instance, CR circuits are used for differentiating or integrating voltage waveforms. This technique is especially important in the processing of TV synchronising signals..

The scanning coils of an electromagnetic CRT can be represented by a series LR circuit. It is therefore important to appreciate the relationship between the applied voltage and the resulting current waveform in this type of circuit if the operation of timebase circuits found in television cameras and monitors are to be fully understood.

- 1.2 You will be expected to have covered the basics of capacitance and inductance on previous courses. The first two chapters of this package should not, therefore, be new to you, but as you may well not have thought about these subjects for some considerable time, these chapters are intended as revision. If you find the material unfamiliar, then the A course programmed texts on Capacitance and Inductance are available from your tutor.

2.

CAPACITANCE REVISION

2.1 Capacitance in a DC circuit

If an emf is applied to a circuit containing an uncharged capacitor a current will flow charging the capacitor. This is illustrated in figure 1.

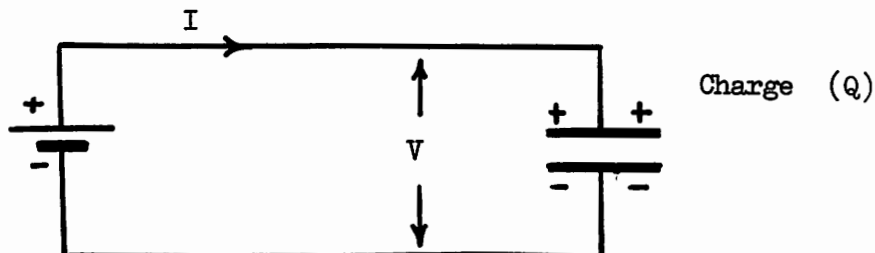


Figure 1 - Charge on a Capacitor

The charge stored on the capacitor after time (t) will be equal to the product of the average current and time.

$$\text{Charge (Q)} = I \times t$$

As the capacitor charges the voltage across it rises. The voltage is given by :

$$V = \frac{Q}{C}$$

When this voltage equals the applied voltage the capacitor is fully charged and the current will have fallen to zero.

- 2.2 It should be noted that in a practical circuit there cannot be an instantaneous change in voltage, or a voltage step, across a capacitor. Sufficient time must elapse to allow the capacitor to charge or discharge before the voltage can change.

2.3 Constant Current Charging

The voltage across a capacitor is given by :

$$V = \frac{Q}{C} \quad \text{where} \quad Q = I \times t$$

Hence,
$$V = \frac{I \times t}{C}$$

If the capacitor is charged with a constant current (I) then it follows that the voltage across the capacitor will rise linearly with time, V being directly proportional to t.

The rate of change of voltage with respect to time is given by $\frac{dV}{dt}$

$$\text{Now, } \frac{dV}{dt} = \frac{V}{t} = \frac{I \times t}{C} \times \frac{1}{t}$$

Hence,
$$\frac{dV}{dt} = \frac{I}{C} \text{ Volts/sec}$$

2.4 Capacitors in Parallel

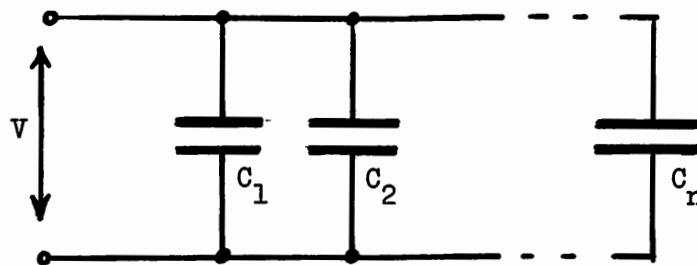


Figure 2 - Capacitors in Parallel

Consider Figure 2 a circuit comprising a number of capacitors in parallel. The total current drawn from the supply will be the sum of the individual charging currents drawn by each capacitor. The total charge stored must therefore be equal to the sum of the charges stored on each of the capacitors.

$$Q_{\text{TOTAL}} = Q_1 + Q_2 + \dots + Q_n$$

Where $Q_1 = C_1 V$, $Q_2 = C_2 V$ etc.

Hence $Q_{\text{TOTAL}} = C_1 V + C_2 V + \dots + C_n V$

Now, total effective capacitance is given by:

$$C_{\text{TOTAL}} = \frac{Q_{\text{TOTAL}}}{V} = \frac{C_1 V + C_2 V + \dots + C_n V}{V}$$

$$C_{\text{TOTAL}} = C_1 + C_2 + \dots + C_n$$

2.5 Capacitors in Series

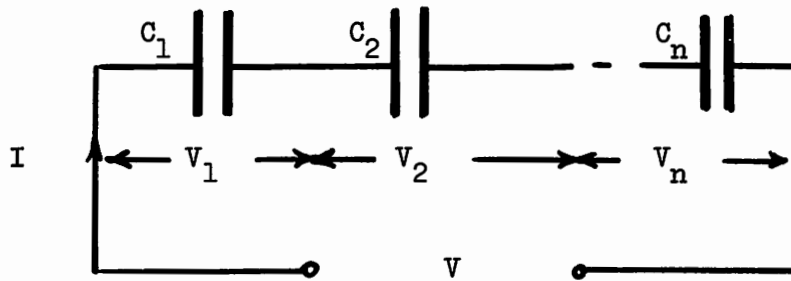


Figure 3 - Capacitors in series

Figure 3 shows a number of capacitors connected in series. The same current flows through each capacitor hence the total effective charge stored by the combination is equal to the charge on any one capacitor.

$$Q_{\text{TOTAL}} = Q_1 = Q_2 \text{ etc.}$$

The applied voltage will equal the sum of the potentials dropped across each of the capacitors.

$$V = V_1 + V_2 + \dots + V_n$$

Where $V_1 = \frac{Q}{C_1}$ $V_2 = \frac{Q}{C_2}$ etc

Hence $V = \frac{Q}{C_1} + \frac{Q}{C_2} + \dots + \frac{Q}{C_n}$

Now the effective capacitance of the combination

$$C_T = \frac{Q}{V} \quad \text{hence} \quad V = \frac{Q}{C_T}$$

Or $\frac{Q}{C_T} = \frac{Q}{C_1} + \frac{Q}{C_2} + \dots + \frac{Q}{C_n}$

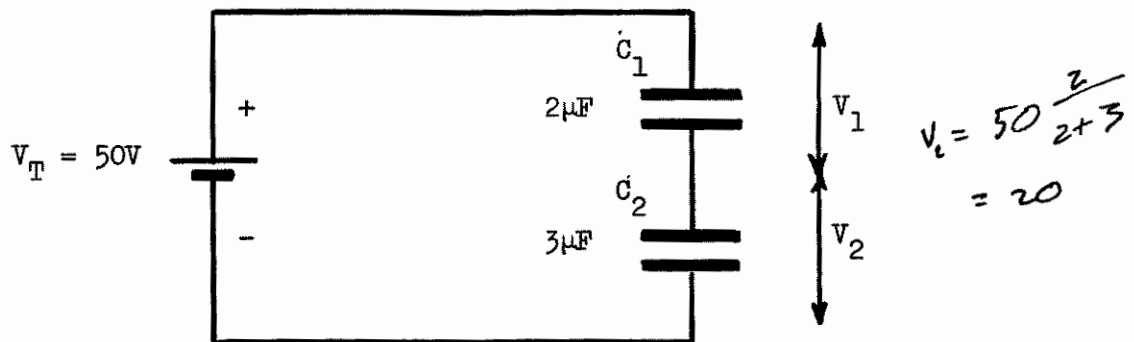
Hence, $\frac{1}{C_{\text{TOTAL}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$

For two capacitors in series this simplifies to

$$C_T = \frac{C_1 C_2}{C_1 + C_2}$$

2.6 Examples

(i)



Calculate the voltage across the $3 \mu F$ (V_2).

We know that the same charging current must flow through both capacitors. Thus, both capacitors must store the same charge.

$$Q_1 = Q_2$$

$$C_1 V_1 = C_2 V_2$$

$$\text{But } (V_1 + V_2) = V_T \quad (V_T = 50 \text{ Volts})$$

$$V_1 = (V_T - V_2)$$

$$\text{Thus } C_1 (V_T - V_2) = C_2 V_2$$

$$C_1 V_T - C_1 V_2 = C_2 V_2$$

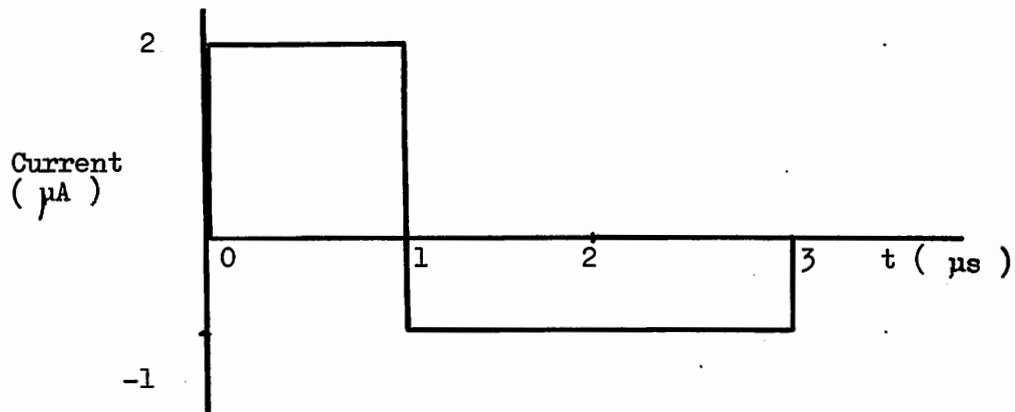
$$\text{giving } V_2 (C_1 + C_2) = C_1 V_T$$

$$\text{Hence } V_2 = \left\{ \frac{C_1}{C_1 + C_2} \right\} V_T$$

$$\text{Thus } V_2 = \left\{ \frac{2}{2+3} \right\} \times 50$$

$$\underline{V_2 = 20 \text{ Volts}}$$

- (ii) If the current through a $1\mu\text{F}$ capacitor varies as shown below, sketch a graph of the voltage across the capacitor versus time.

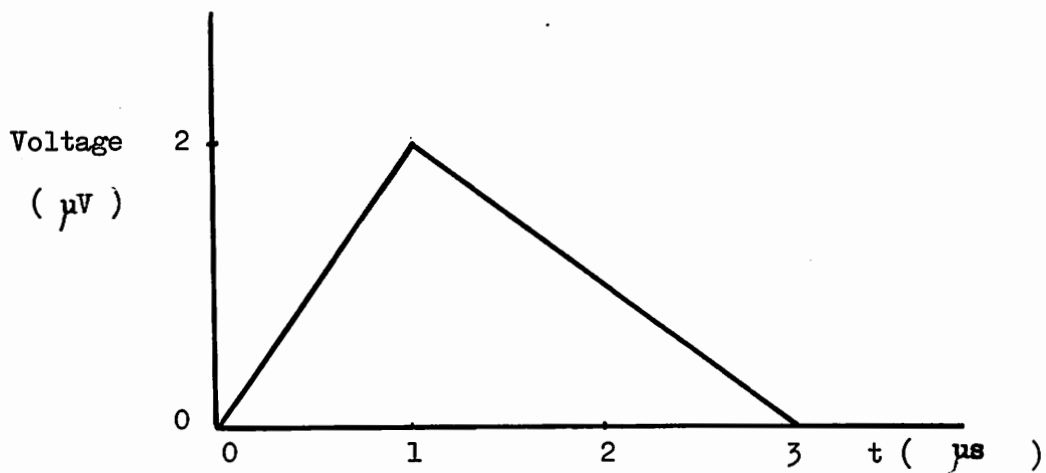


Using $\frac{dV}{dt} = \frac{I}{C}$

For $0 < t < 1$ $I = + 2 \mu\text{A}$
Hence $\frac{dV}{dt} = \frac{2 \times 10^{-6}}{1 \times 10^{-6}} = + 2 \text{ Volts/sec}$

For $1 < t < 3$ $I = - 1 \mu\text{A}$
And $\frac{dV}{dt} = \frac{- 1 \times 10^{-6}}{1 \times 10^{-6}} = -1 \text{ Volt/sec}$

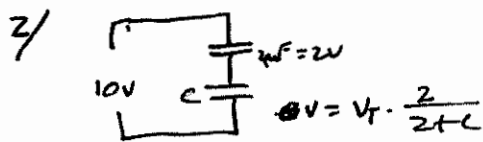
If $V = 0$ when $t = 0$ we can sketch the graph of V versus t below:



2.7 Exercise

1. A sawtooth waveform is generated by charging a capacitor with a constant current of 10mA for 50μs and then discharging it quickly. Determine a suitable value for the capacitor if the waveform is to have an amplitude of 50V.
2. A potential divider comprises two capacitors in series across a 10V d.c. supply. If the p.d. across the 2μF capacitor is 2V determine the value of the other capacitor.
3. A 1μF capacitor is charged to 100V and is then connected across a circuit comprising a 2μF capacitor and a 3μF capacitor in series. Determine the p.d. across each of the 3 capacitors.

$$1/ \quad \frac{dV}{dt} = \frac{I}{C} \quad \frac{50}{50\mu s} = \frac{10 \times 10^{-3}}{C} \quad C = \frac{10 \times 10^{-3} \cdot 50 \mu s}{50} = 10 \mu F.$$



$$V_{2\mu F} = 10 \times \frac{2}{2+C} \quad \frac{2}{10} = \frac{2}{2+C}$$

$$2 = 10 \times \frac{2}{2+C} \quad \cdot 2 = \frac{C}{2+C}$$

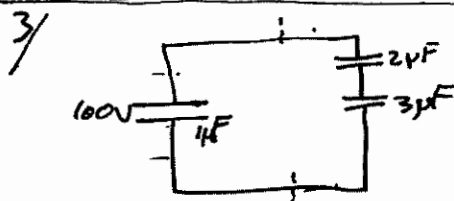
$$\cdot 2(2+C) = C$$

$$\cdot 4 + \cdot 2C = C$$

$$\cdot 4 = C - \cdot 2C$$

$$\cdot 4 = \cdot 8C$$

$$C = \frac{4}{8} = \frac{1}{2} \mu F$$



$$V_{\mu F} = 100 \times \frac{1\mu F}{2.2}$$

$$= 45 \text{ Volts}$$

$$\frac{1}{C} = \frac{1}{2} + \frac{1}{3}$$

$$C = 1.2 \mu F$$

$$C_{Tot} = 1.2 + 1 = 2.2 \mu F$$

$$V_{2+3\mu F} = 45 \text{ Volts}$$

$$V_{2\mu F} = 45 \times \frac{3\mu F}{2+3}$$

$$= 27 \text{ Volts}$$

$$V_{3\mu F} = 45 \times \frac{2}{2+3}$$

$$= 18 \text{ Volts}$$

ANSWERS

1. 10nF
2. $0.5\mu\text{F}$
3. $1\mu\text{F}$ - 45.5V
 $2\mu\text{F}$ - 27.3V
 $3\mu\text{F}$ - 18.2V

3.

INDUCTANCE - REVISION

3.1 Inductance in a DC circuit

Whereas a capacitor in a circuit tends to keep the voltage constant, inductance tends to maintain a constant current through the circuit.

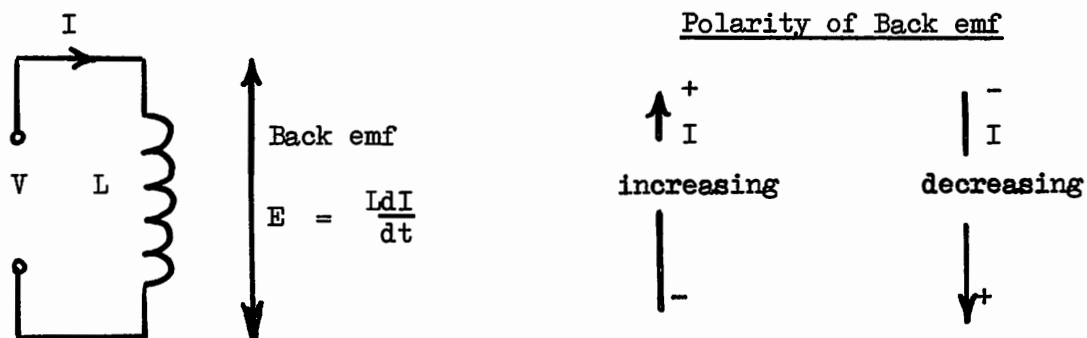


Figure 4 - Inductance and back emf

Consider the circuit of figure 4. If the current through the inductor changes then a back emf will be induced in it. The polarity of this emf will be in such a direction that it always opposes the applied voltage, the voltage causing the current to change.

From Faraday's Law of electromagnetic induction the back emf is given by:

$$E = L \frac{dI}{dt}$$

3.2 Note that if the current remains constant no voltage will be developed across the inductor. If the current increases at a constant rate the back emf will have a constant value. As the back emf is constant and opposes the applied emf, it follows that applying a constant dc voltage across a pure inductance will cause the current through it to rise at a constant rate.

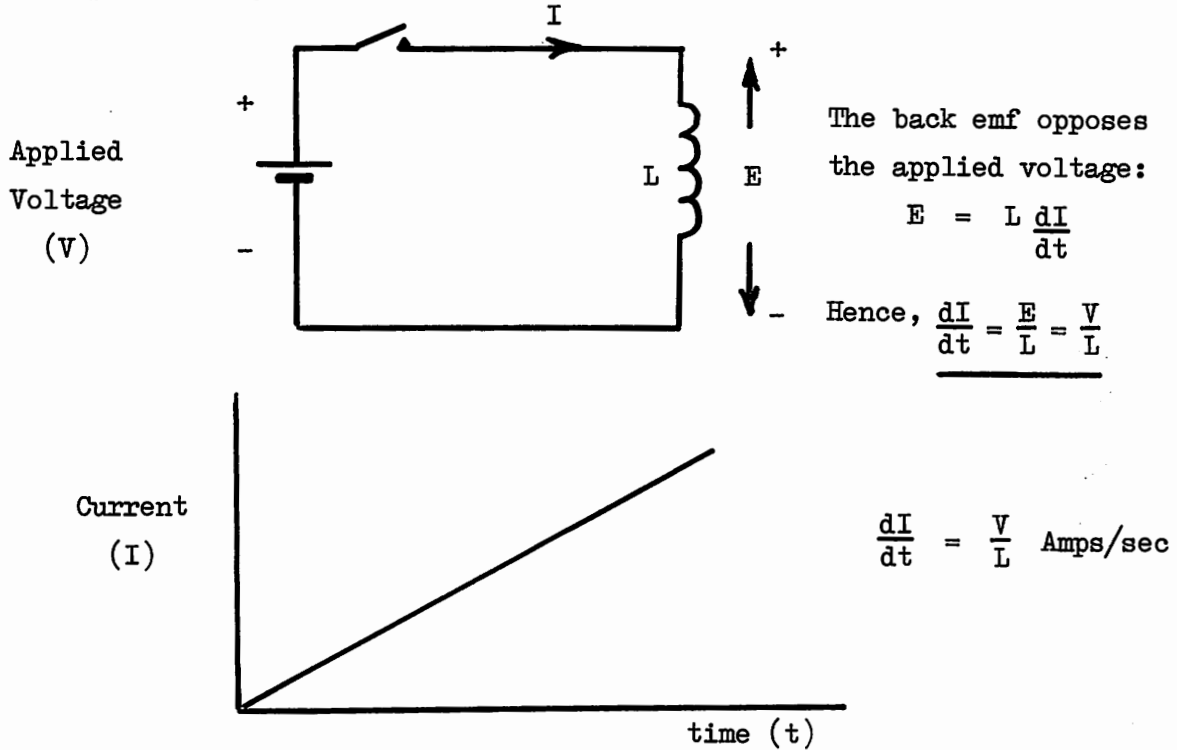
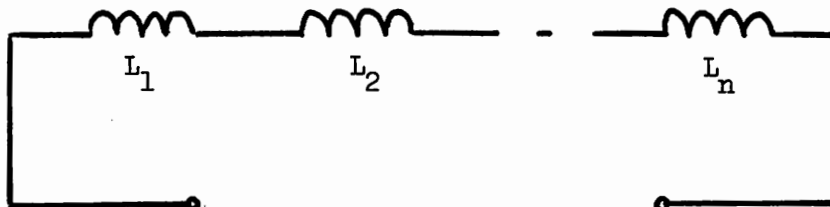


Figure 5 - Applying a constant dc voltage across a pure inductance

3.3 Note that a practical inductor or coil has both inductance and resistance, whereas the above considers the component to be a pure inductance.

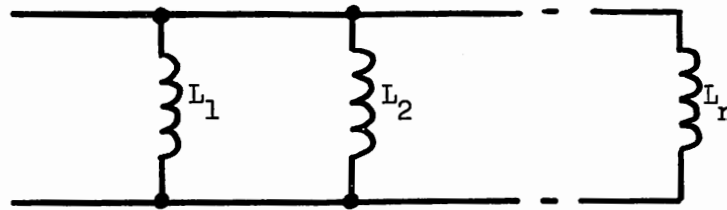
3.4 Inductance in Series



The Total Inductance,

$$L_T = L_1 + L_2 + \dots + L_n$$

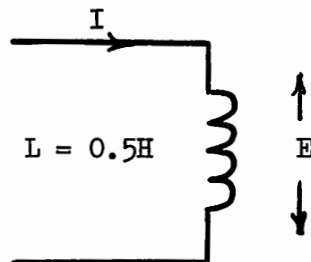
3.5 Inductors in Parallel



$$\frac{1}{\text{Total Inductance } (L_T)} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_n}$$

3.6 Examples

- (i) The current through a pure inductance of value 0.5 H starts from zero and rises linearly for 10 secs reaching a final value of 1 Amp. Sketch a graph of the voltage across the inductor against time.



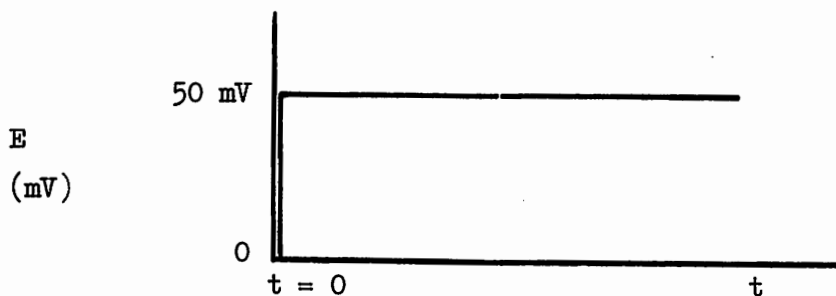
Now, $\frac{\Delta I}{\Delta t} = \frac{1}{10} \text{ Amps/sec}$

i.e. $\frac{dI}{dt} = 0.1 \text{ A/sec}$

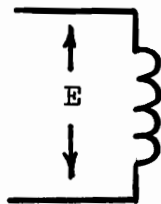
Using $E = L \frac{dI}{dt}$

Then, $E = 0.5 \times 0.1$

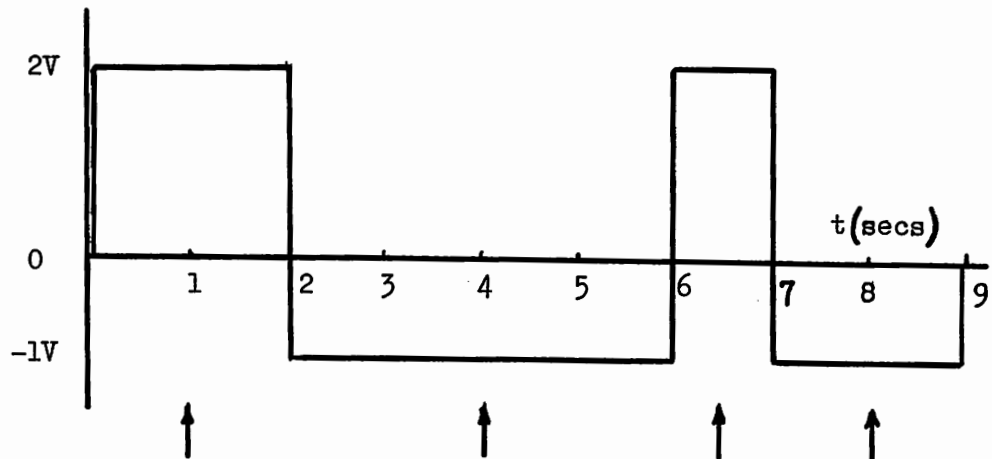
$E = 50 \text{ mV}$



- (ii) Sketch a current waveform which would produce the following self induced emf in an inductor of 2 H.



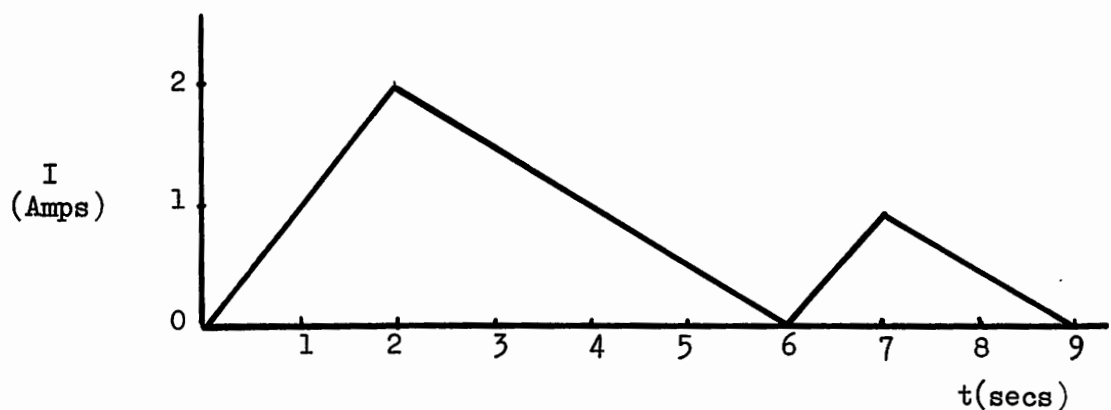
$$L = 2 \text{ H}$$



Using $E = L \frac{dI}{dt}$

Then, $\frac{dI}{dt} = \frac{E}{L}$ hence $\frac{dI}{dt} = \frac{2}{2} = +1 \text{ A/s}$ $\frac{dI}{dt} = \frac{-1}{2} = -0.5 \text{ A/s}$ $\frac{dI}{dt} = \frac{2}{2} = +1 \text{ A/s}$ $\frac{dI}{dt} = \frac{-1}{2} = -0.5 \text{ A/s}$

If $I = 0$ at $t = 0$ then the current waveform required will be:



3.7 Exercises

Two coils of negligible resistance have an inductance of 50mH and 100mH respectively. If the coils are connected,

a) in series

b) in parallel

across a 100V d.c. supply how long will it take for the supply current to reach 1A.

$$L = 50\text{mH} + 100\text{mH}$$

$$\text{Series } L = 150\text{mH}$$

$$E = L \frac{dI}{dt} \quad \begin{array}{l} E = 100 \\ I = 1 \end{array}$$

$$\frac{dI}{dt} = \frac{E}{L}$$

$$\frac{1}{dt} = \frac{E}{L dI} = \frac{100}{150\text{mH} \cdot 1}$$

$$\therefore dt = \underline{\underline{1.5\text{mS}}}$$

$$\begin{aligned} // L &= \frac{50 \cdot 100}{150} \\ &= 33.3 \end{aligned}$$

$$\begin{aligned} \therefore \frac{1}{dt} &= \frac{100}{33.3 \cdot 1} \\ &= \underline{\underline{333\text{mS}}} \end{aligned}$$

ANSWERS

- a. 1.5 ms
- b. 0.33 ms

4. SERIES CR AND SERIES LR CIRCUITS

In the following chapters we are going to consider in detail the voltage and current waveforms associated with applying:

- a) A single current step to a series CR circuit.
- b) A triangular current waveform to a series LR circuit.
- c) Repetitive, rectangular voltage waveforms to both series CR circuits and series LR circuits.

4.1 A current step applied to a series CR circuit

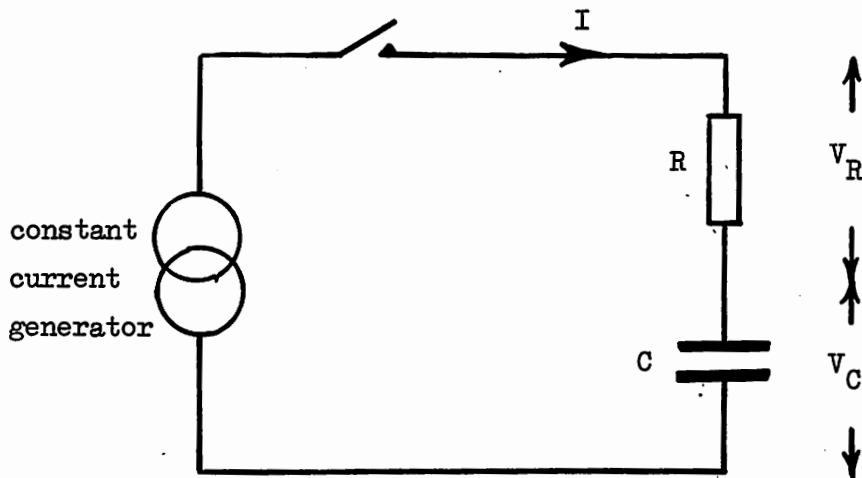


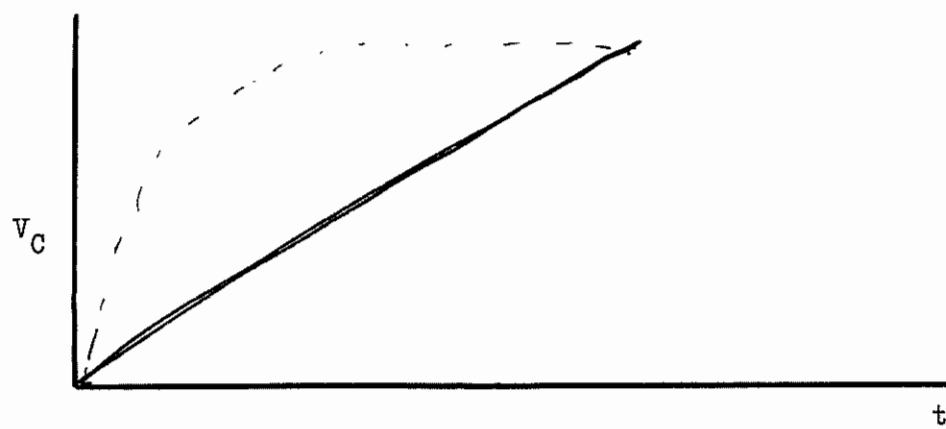
Figure 6 - Series CR circuit

When the switch is closed a constant current is passed through the resistor and the capacitor.

In paragraph 2.3 we considered the effect of charging a capacitor with a constant current. Sketch, on the next page, how you would expect

- a) The voltage (V_C) across the capacitor to vary with time.
- b) The voltage (V_R) across the resistor to vary with time.

Assume that the capacitor is initially discharged.



ANSWER

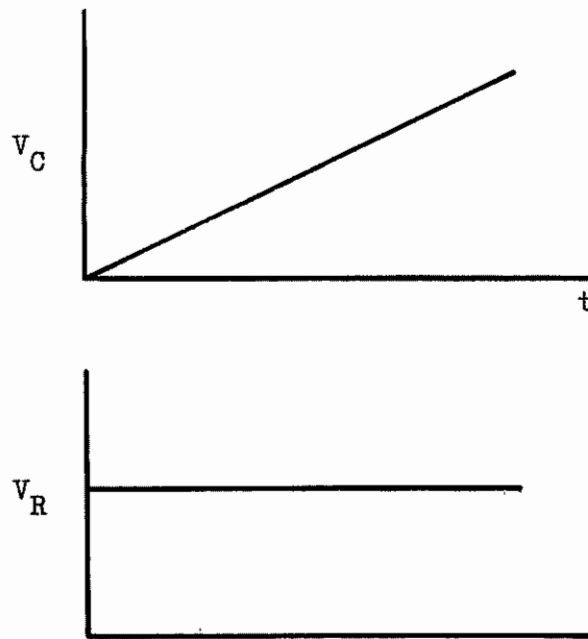


Figure 7 - V_C and V_R

- c) What determines the slope of the V_C graph?

Current

$$\frac{dV}{dt} = \frac{I}{C}$$

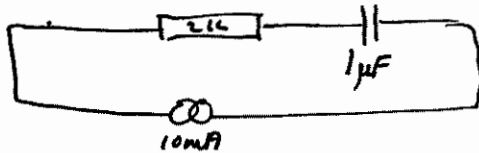
- d) What determines the magnitude of V_R ?

$$V = IR$$

$\therefore I$ determines

4.2 Exercise

- a. A circuit comprises a 2k resistor in series with a 1 μ F capacitor. Sketch graphs of V_C and V_R versus time if a constant current of 10 mA flows through the circuit for a period of 5 ms.
- b. Sketch the supply voltage waveform required to enable this current to flow.



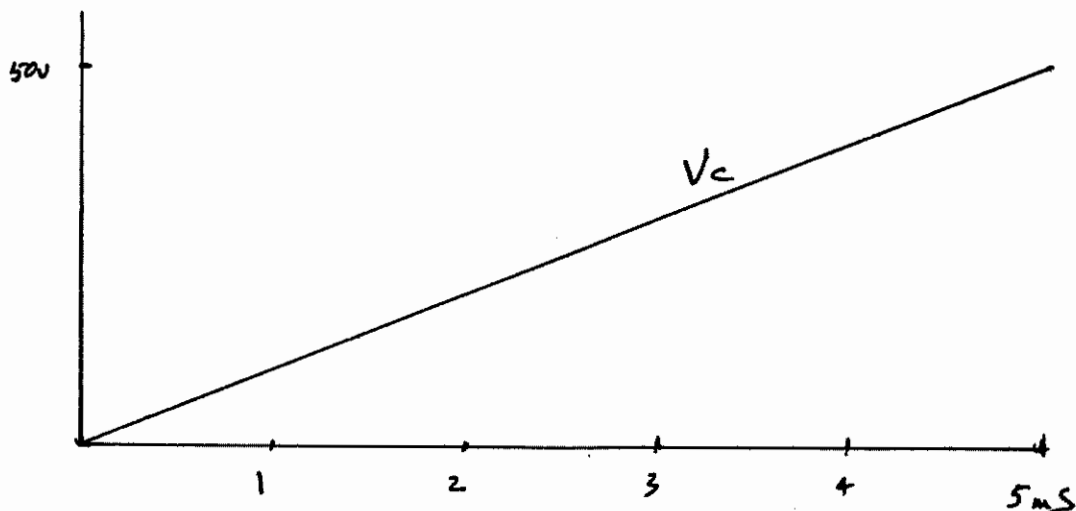
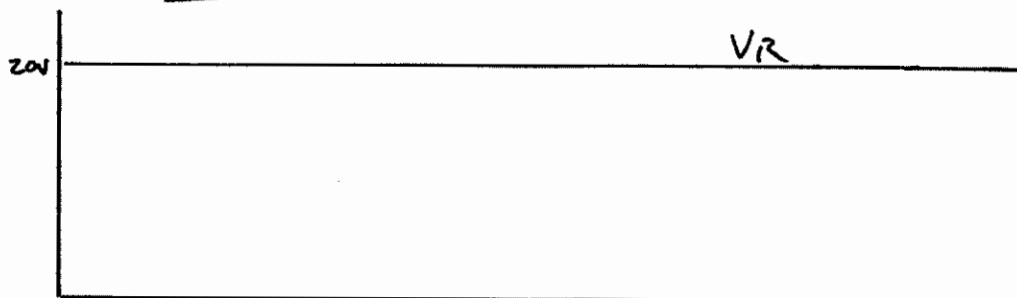
$$\begin{aligned} V_R &= IR \\ &= 10\text{mA} \cdot 2k \\ &= \underline{\underline{20\text{Volts}}} \end{aligned}$$

$$\frac{dV_C}{dt} = \frac{I}{C}$$

$$\Delta V = \frac{I \Delta t}{C}$$

$$= \frac{10\text{mA} \cdot 5\text{ms}}{1 \times 10^{-6}}$$

$$= \underline{\underline{50\text{Volts after } 5\text{ms}}}$$

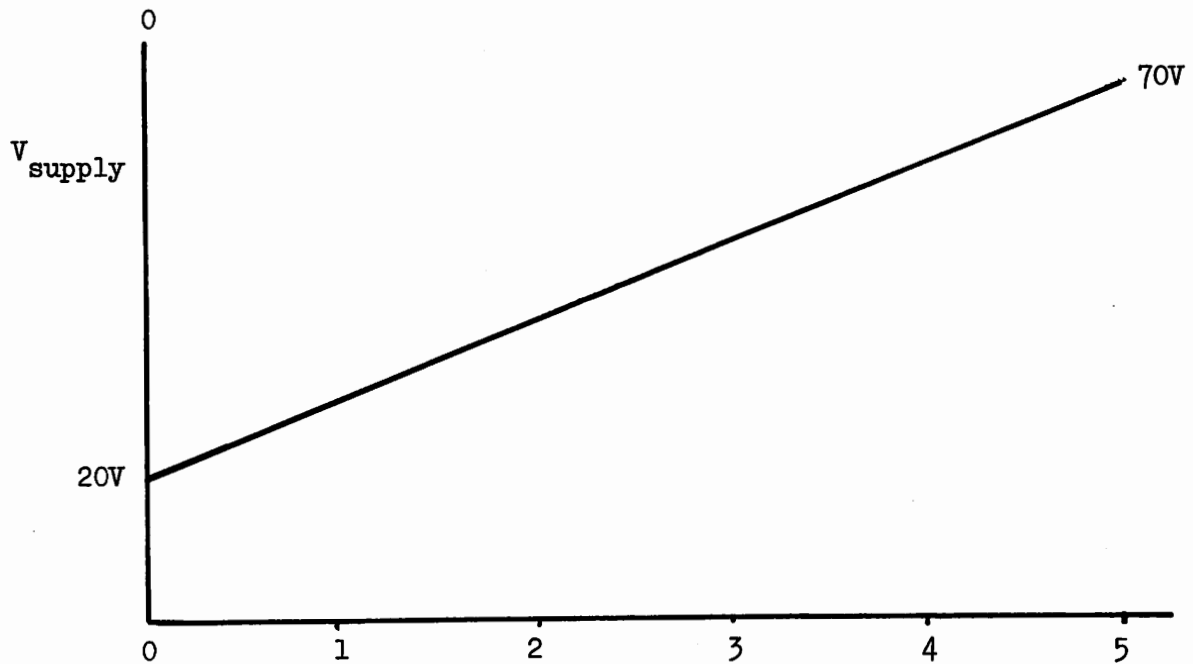
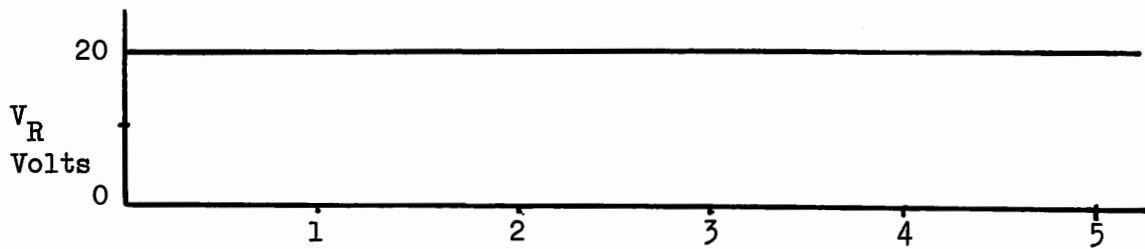
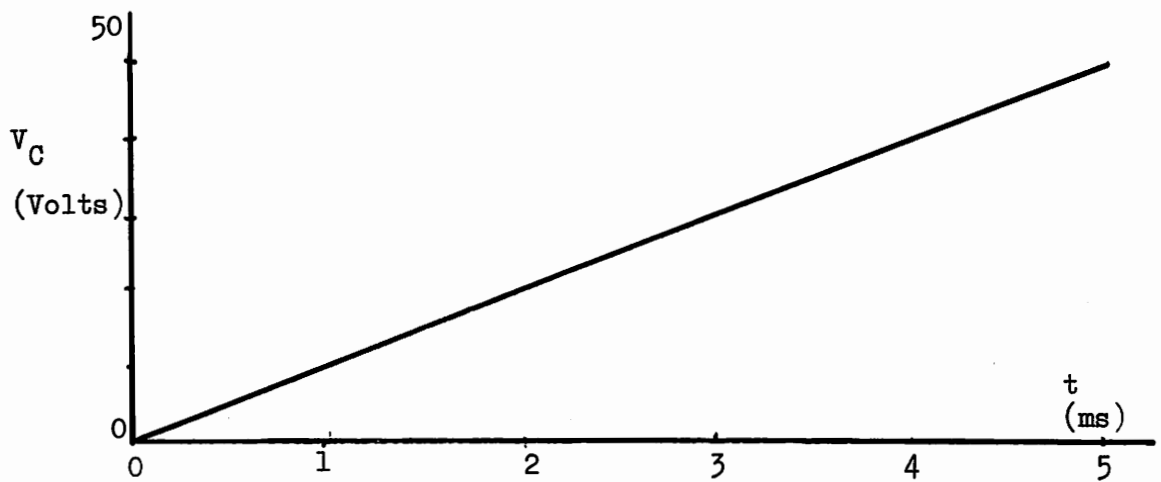


ANSWERS TO PARAGRAPH 4.1

$$\frac{dv_C}{dt} = \frac{I}{C} \text{ volts/second}$$

$$V_R = IR \text{ volts.}$$

ANSWERS TO PARAGRAPH 4.2



5.

A TRIANGULAR CURRENT WAVEFORM APPLIED
TO A SERIES LR CIRCUIT

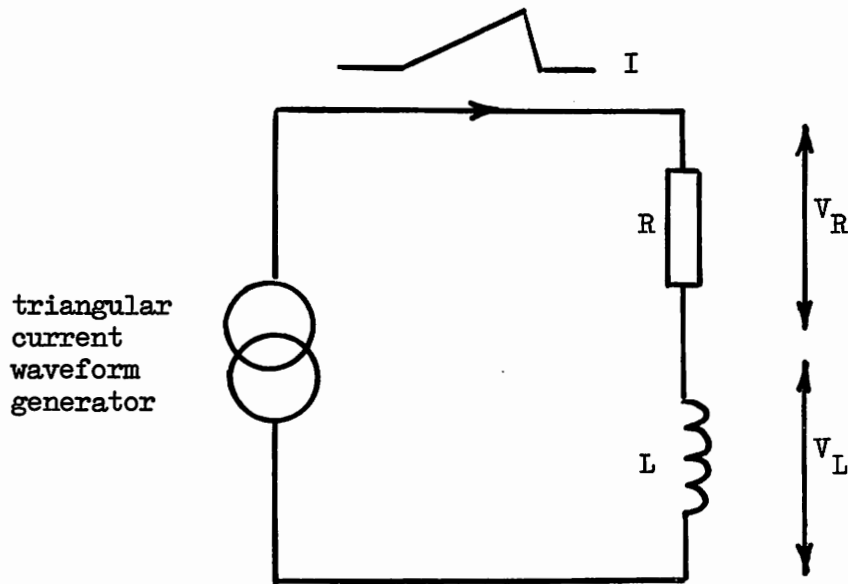


Figure 8 - Series LR circuit

The circuit in figure 8 could represent the line scan coils of a television monitor. For the timebase to function correctly the current through the coils is required to rise linearly and fall to zero quickly. An example of this is shown in figure 9.

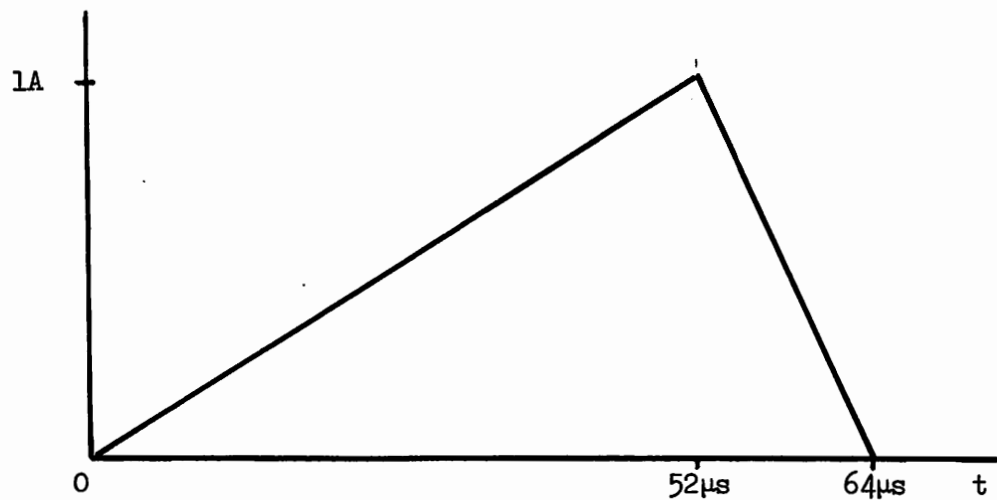


Figure 9 - Line timebase current waveform

5.1 What determines the magnitude and shape of

a) The voltage waveform across R?

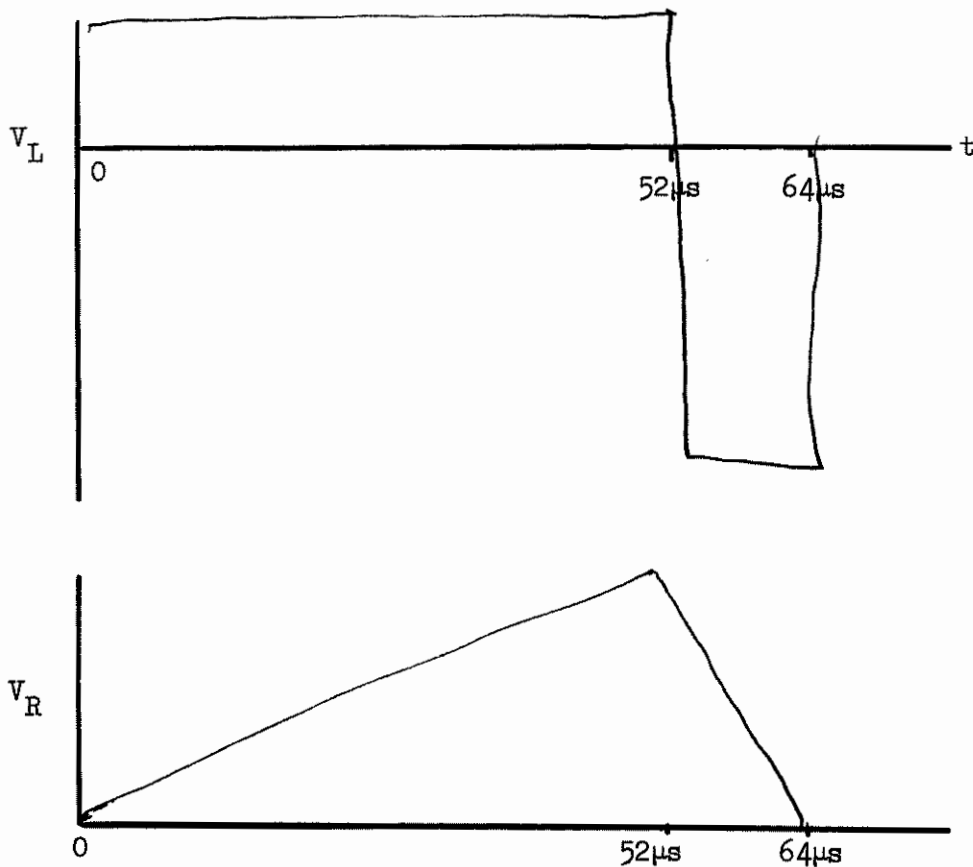
Current I_R

b) The voltage waveform across L?

$$E = L \frac{dI}{dt}$$

If you are not sure then check paragraph 3.1 where the fundamentals of this were considered.

5.2 Sketch below the shape of V_L and V_R if the current waveform is as shown in figure 9.



- 5.3 Determine values for the maximum and minimum voltages of V_L and V_R if the scan coils represented by the circuit of figure 8 have a resistance of 75Ω and an inductance of 10mH .

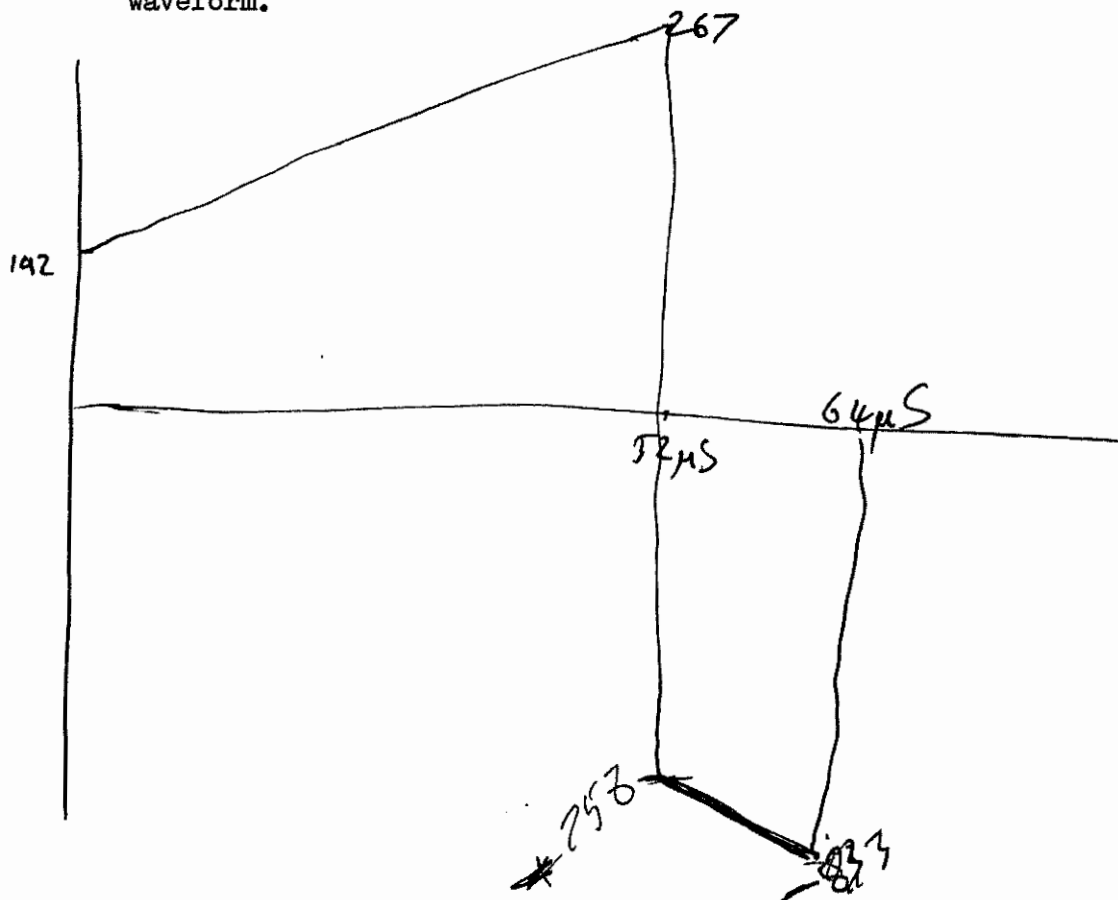
$$\begin{aligned} \text{Max } V_L &= L \frac{dI}{dt} \\ &= 10\text{mH} \cdot \frac{1}{52\mu\text{s}} \\ &= 192 \text{ volts} \end{aligned}$$

$$\begin{aligned} \text{Min } V_L &= L \frac{dI}{dt} \\ &= 10\text{mH} \cdot \frac{1}{12\mu\text{s}} \\ &= 833 \text{ volts} \end{aligned}$$

$$\begin{aligned} V_{R\text{Max}} &= 1.75 \\ &= 75 \text{ volts} \end{aligned}$$

$$\begin{aligned} V_{R\text{Min}} &= 0.75 \\ &= 0 \text{ volts} \end{aligned}$$

- 5.4 Sketch the voltage waveform required to drive the current waveform of figure 9 through the coils. Determine the magnitude of the waveform.



ANSWERS TO PARAGRAPH 5.1

(a) $V_R = I \times R$

V_R is directly proportional to I .

(b) $V_L = L \frac{dI}{dt}$

V_L is constant while $\frac{dI}{dt}$ is constant.

V_L will be positive while the current is rising and negative while the current is falling.

ANSWERS TO PARAGRAPHS 5.3 and 5.4

(a) $V_R = IR$ hence V_R maximum when I maximum

$$V_R (\text{max}) = 1 \times 75 = 75V$$

(b) $V_L = L \frac{dI}{dt}$

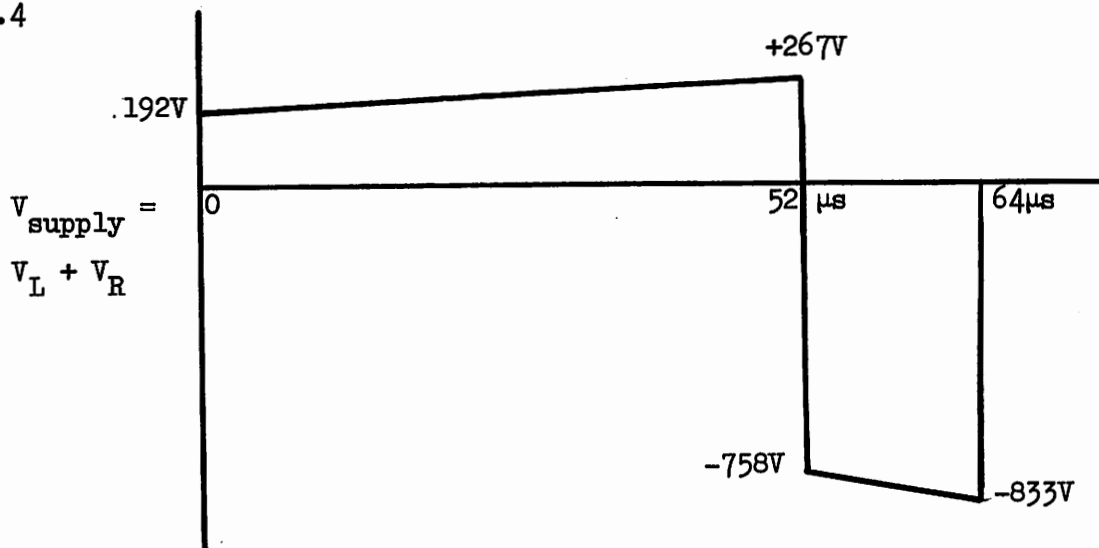
during period 0 to $52\mu s$

$$V_L = 10 \times 10^{-3} \times \frac{1}{52 \times 10^{-6}} = +192V$$

during period $52\mu s$ to $64\mu s$

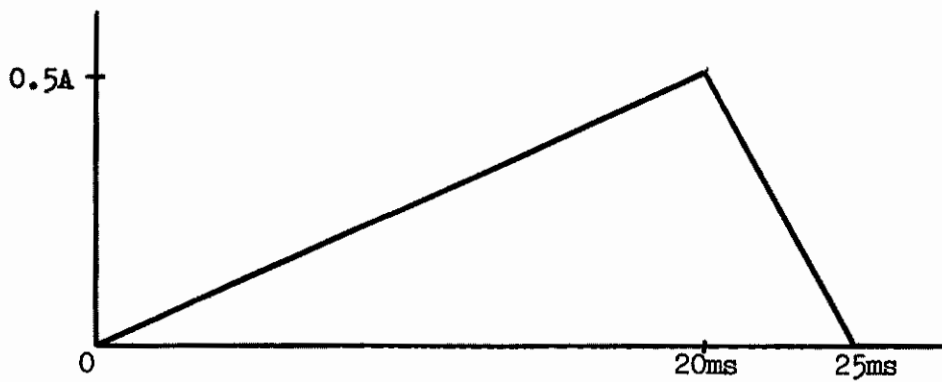
$$V_L = \frac{-10 \times 10^{-3}}{12 \times 10^{-6}} = -833V$$

5.4



5.5 Exercise

- a. It is required to drive the following current through a coil of 100mH and 10Ω.



What voltage waveform must be applied to the circuit?

$$V_L = L \frac{dI}{dt}$$

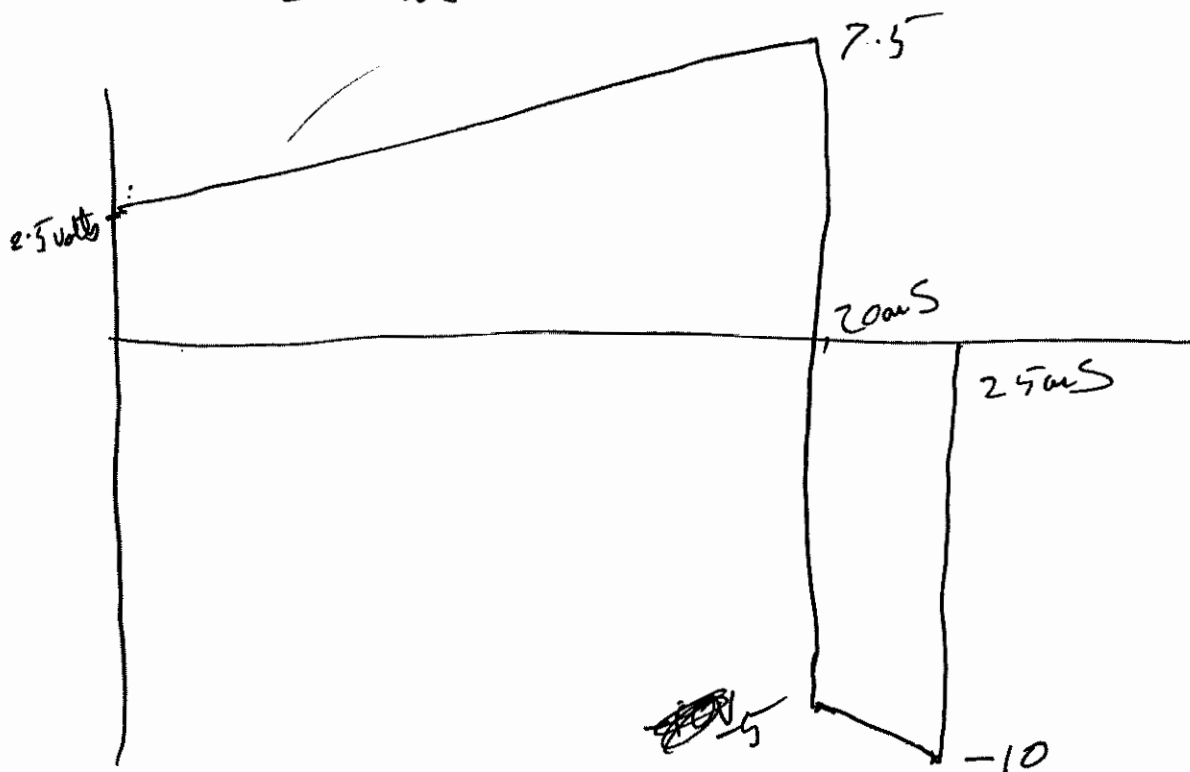
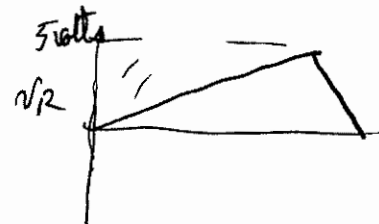
$$V_L = 100\text{mH} \cdot \frac{0.5}{20\text{ms}}$$

$$= 2.5\text{ Volts}$$

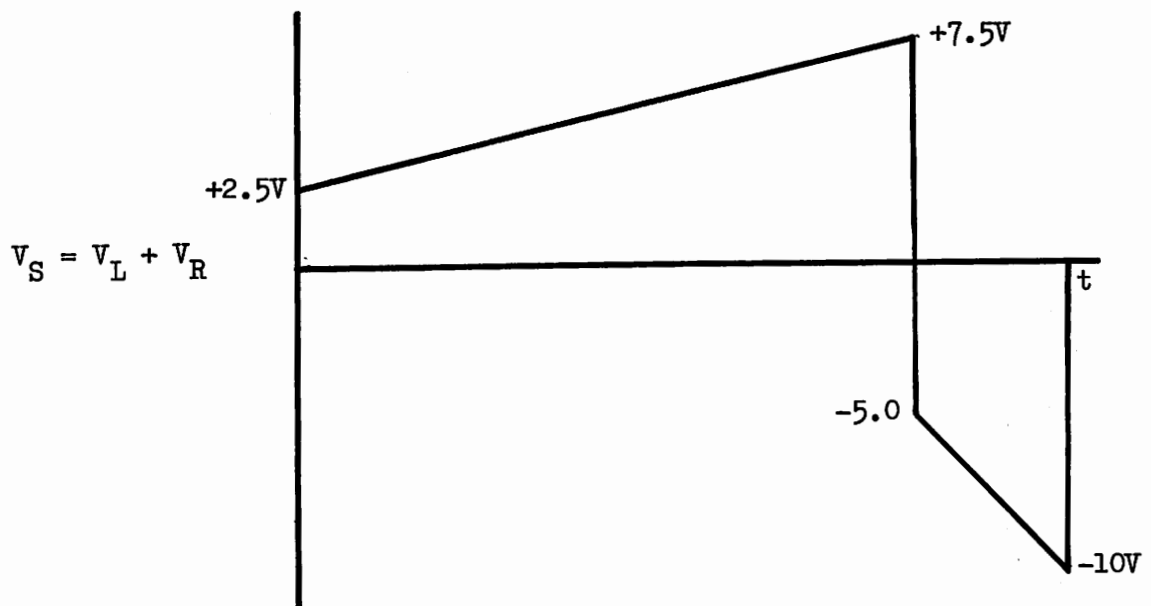
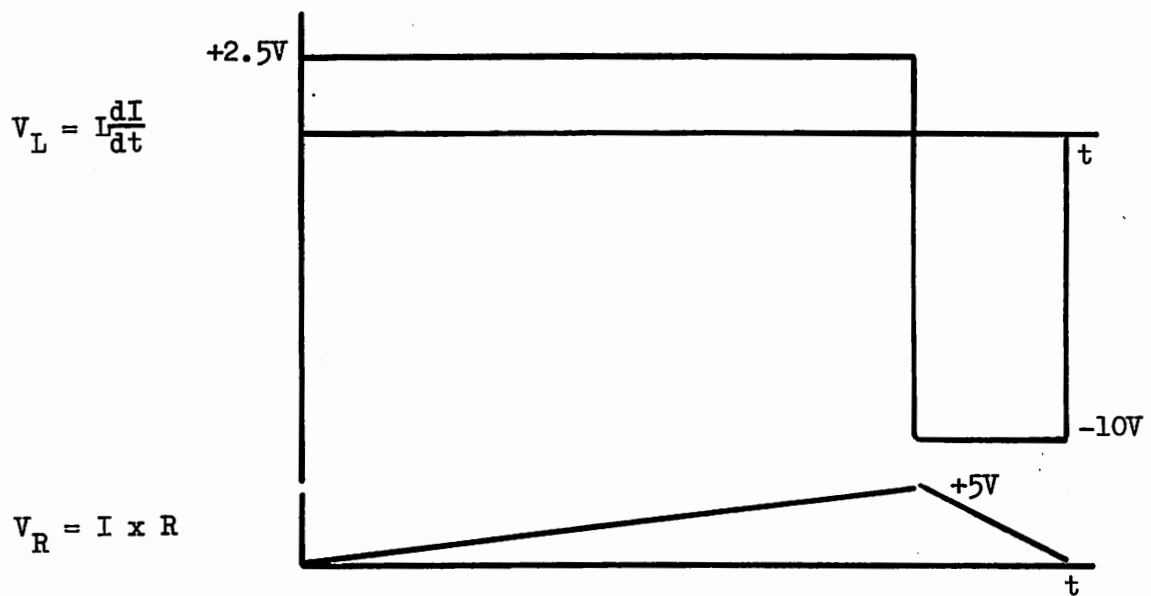
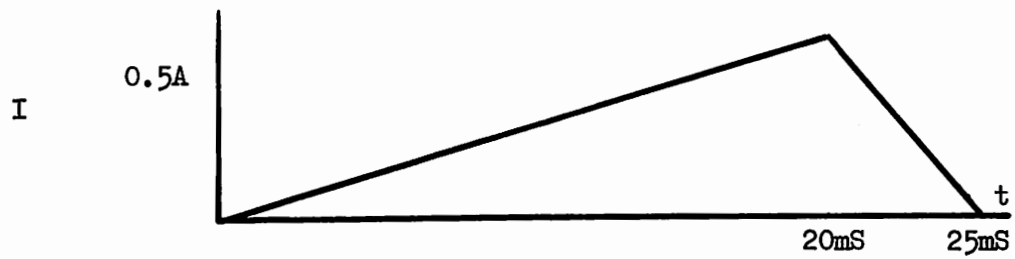
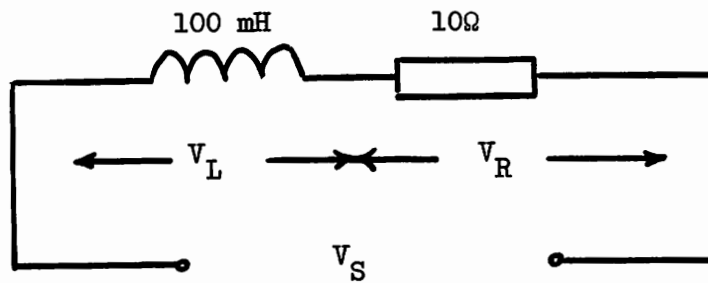
$$V_R = I \cdot R$$

$$= 0.5 \cdot 10$$

$$= 5\text{ Volts}$$



ANSWER TO PARAGRAPH 5.5



6.

A VOLTAGE STEP APPLIED TO A SERIES CR CIRCUIT

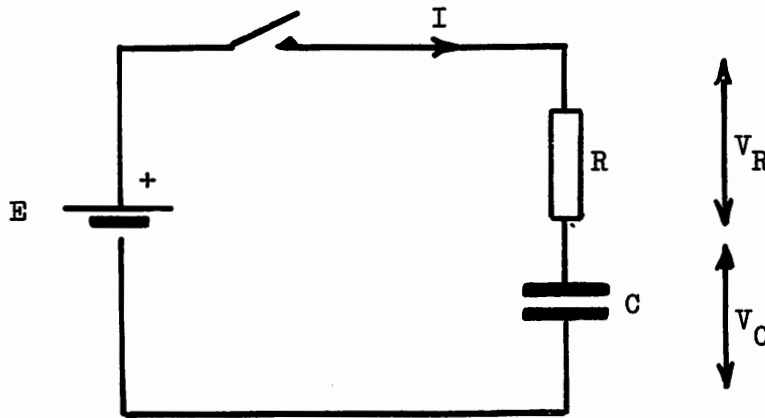


Figure 9a -Series CR circuit

Consider the circuit of Figure 9a, what will happen to the current in the circuit and the charge on the capacitor when the switch is closed?

As current starts to flow the capacitor will begin to charge-up. The current through the circuit will be

$$I = \frac{V_R}{R} \text{ where } V_R = E - V_C$$

$$\text{therefore } I = \frac{E - V_C}{R}$$

This means that as the capacitor charges up, and V_C rises, the current flowing in the capacitor (I) will reduce.

6.1 In paragraph 2.3 we found that the current charging a capacitor determines the rate of change of voltage across it

$$\frac{dV_C}{dt} = \frac{I}{C} \text{ volts/sec}$$

From this it follows that initially, the capacitor charges quickly because

$$\frac{dV_C}{dt} = \frac{I}{C} = \frac{E - V_C}{CR} \text{ where } V_C = 0$$

$$\text{hence } \frac{dV_C}{dt} = \frac{E}{CR} \text{ volts/second}$$

but as the capacitor charges V_C rises and $\frac{dV_C}{dt}$ will fall.

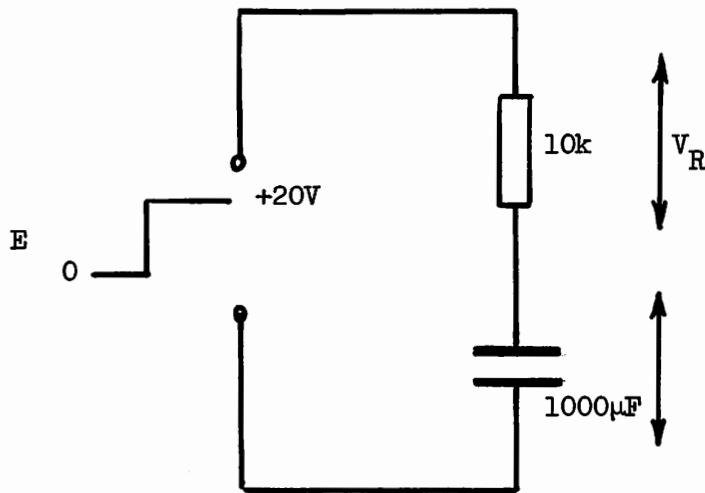


Figure 10 - Series CR Circuit

6.2 Consider the circuit shown in figure 10, in which a 20V step is applied to a series CR circuit.

At the instant the step is applied the voltage across the capacitor cannot change, hence the p.d. across the resistor will rise to +20V. The current will be

$$I = \frac{V_R}{R} = \frac{20}{10k} = 2mA$$

If we say this current flows for say, 5 seconds, the change in the charge on the capacitor will be

$$\Delta Q = I \times t$$

and the change in voltage on the capacitor will be

$$\Delta V_C = \frac{\Delta Q}{C} = \frac{I \times t}{C} = \frac{2 \times 10^{-3} \times 5}{10^{-3}} = 10V$$

Now if the voltage on the capacitor has risen to 10V whilst the input remains at +20V it follows that the pd across the resistor must have fallen to

$$20V - 10V = 10V$$

This means that the capacitor charging current will have fallen to

$$\frac{V_R}{R} = \frac{10}{10k} = 1mA$$

Once again if we say that this current flows for a further five seconds the voltage on the capacitor will rise by

$$\Delta V_C = \frac{I \times t}{C} = \frac{10^{-3} \times 5}{10^{-3}} = 5V$$

Therefore after a total of 10 seconds the voltage on the capacitor will have risen to

$$10V + 5V = 15V$$

6.3 Using the same technique determine the voltage on the capacitor after a total time of 15 seconds and 20 seconds.

$$V_C = 15 \quad \therefore V_R = 20 - 15 = 5$$

$$\begin{aligned} \therefore I &= \frac{5}{10k} \\ &= \frac{1}{2} \text{mA} \end{aligned}$$

$$\begin{aligned} \Delta V_C &= \frac{I \cdot t}{C} = \frac{.5 \text{mA} \cdot 5}{10^{-3}} \\ &= 2.5 \text{ volts} \end{aligned}$$

$$\therefore V_{C \text{ total}} = 15 + 2.5 = 17.5 \quad (15 \text{ Secs})$$

$$V_C = 17.5 \quad V_R = 20 - 17.5 = 2.5$$

$$\begin{aligned} \therefore I_R &= \frac{2.5}{10k} \\ &= .25 \text{mA} \end{aligned}$$

$$\begin{aligned} \Delta V_C &= \frac{I \cdot t}{C} \\ &= \frac{.25 \text{mA} \cdot 5}{10^{-3}} \\ &= 1.25 \end{aligned}$$

$$\therefore V_C = 17.5 + 1.25 = 18.75 \quad (20 \text{ Secs})$$

6.4 Now plot a graph of V_C against time from 0 to 20 seconds drawing a smooth curve between the points.

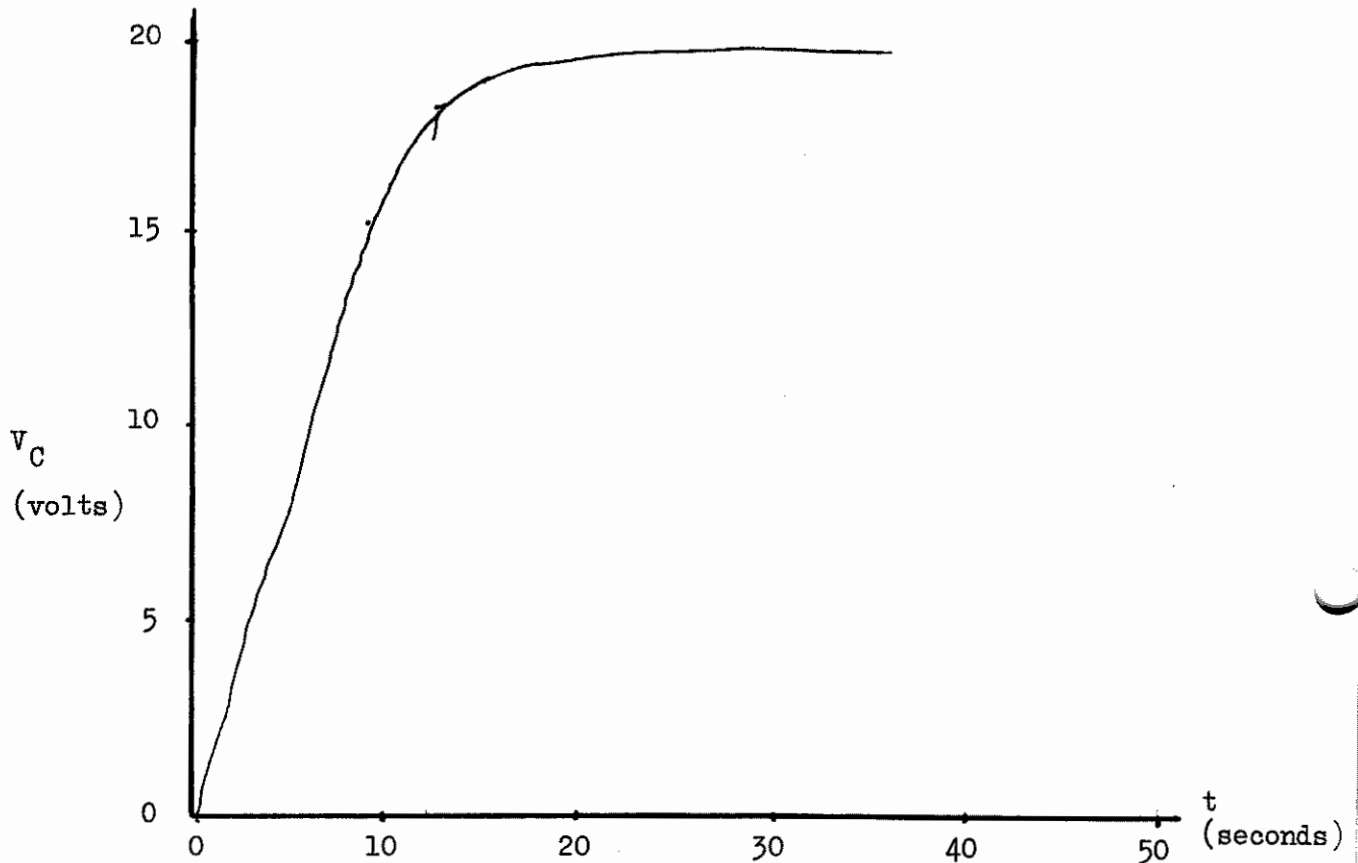


Figure 11 - Graph of V_C against time

You should be able to see from the curve that its slope reduces with increase in time and is exponential in shape.

- 6.5
- Construct the circuit of figure 10 on a breadboard.
 - Ensure the capacitor is initially discharged.
 - With the power unit not connected to the circuit set its output to +20V.
 - Connect the power unit to the circuit and measure, using an AVO, the voltage across the capacitor after an interval of 10, 20, 30, 40 and 50 seconds. Record your readings in the table over.

time (secs)	V_C
0	0 0
10	~~10~~ 11
20	~~15~~ 16
30	~~16~~ 17
40	18.5
50	19

- e. Use the above table to plot a second graph on the axes given in figure 11.
- f. What explanation can you give for the difference between the two graphs.

- 6.6 The discrepancies must be due to the fact that the first method was an approximation. It assumed that the charging current remained constant during each time period, whereas in reality it would be varying continuously.
- 6.7 A more accurate method of determining the shape of the curve is to use one involving calculus.

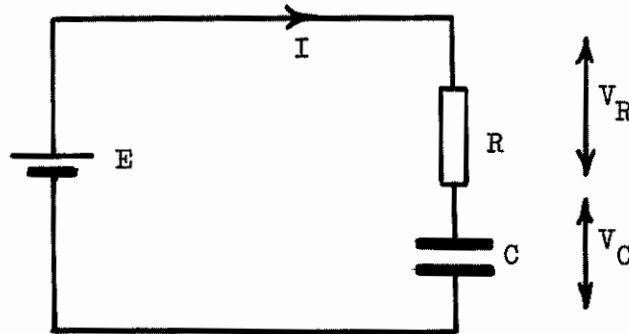


Figure 12 - Series CR Circuit.

Consider figure 12.

We know that the supply voltage

$$E = V_R + V_C \quad \text{where} \quad V_R = IR$$

$$\text{therefore } E = IR + V_C$$

$$\text{now from page 5} \quad \frac{dV_C}{dt} = \frac{I}{C}$$

$$\text{or} \quad I = C \frac{dV_C}{dt}$$

therefore the above equation can be rewritten as

$$E = CR \frac{dV_C}{dt} + V_C$$

This is known as a differential equation and can be solved for V_C .

If we assume $V_C = 0$ at $t = 0$ then

$$V_C = E \left(1 - \exp^{\frac{-t}{CR}} \right)$$

after a time t seconds

$$CR = 10$$

$$= 20 \left(1 - e^{\frac{-t}{10}} \right)$$

- 6.8 Use this equation to calculate the voltage across the capacitor in figure 10, when 10, 20, 30, 40 and 50 seconds have elapsed after the application of the voltage step. Insert your answers in the table below.

$t(s)$	V_C
0	
10	12.64
20	17.29
30	19.00
40	19.63
50	19.86

- 6.9 How do these values compare with the values you measured in paragraph 6.5?

More accurate!!

- 6.10 We now have an expression for V_C . Let us now consider the voltage across the resistor.

Now, $V_R = E - V_C$

where $V_C = E (1 - \exp \frac{-t}{CR}) = E - E \exp \frac{-t}{CR}$

hence $V_R = E - (E - E \exp \frac{-t}{CR})$

$$V_R = E \exp \frac{-t}{CR}$$

after a time t seconds

6.11 Use the equation given above to calculate the voltage across the resistor in figure 10 after 10, 20, 30, 40 and 50 seconds have elapsed after application of the voltage step. Insert the values calculated directly on figure 13 and hence obtain a graph of V_R against time.

$$V_R = V e^{-\frac{t}{CR}}$$

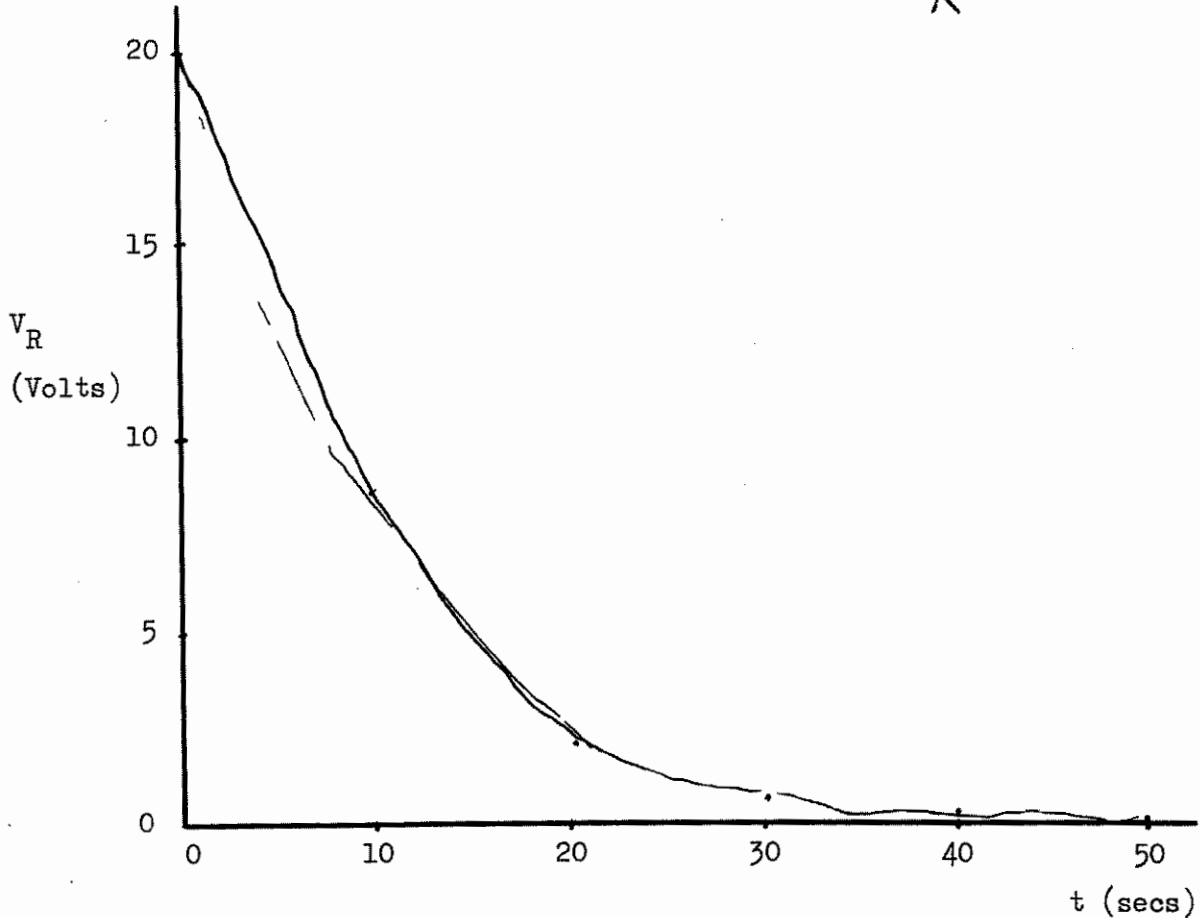


Figure 13 - Plot of V_R against time

6.12 Graphs can also be drawn for the discharge of a capacitor through a resistor.

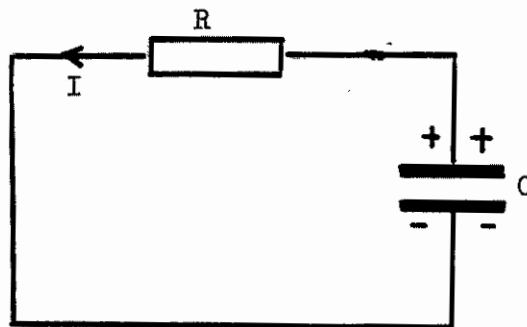


Figure 14 - Discharge of a Capacitor

In this case, the voltage across the capacitor starts at V and falls exponentially to zero. A plot of V_C against time will have the same shape as figure 13 and hence the voltage on the capacitor will be given by:

$$V_C = V \exp \frac{-t}{CR}$$

discharge!!

where V is the initial voltage on the capacitor

7.

THE TIME CONSTANT OF A CR CIRCUIT

The time constant of a series CR circuit can be calculated from

$$\text{Time constant} = C \times R \text{ seconds}$$

If we consider the equation for the charge of a capacitor

$$V_C = E \left(1 - \exp\left(\frac{-t}{CR}\right) \right)$$

or the discharge of a capacitor

$$V_C = V \exp\left(\frac{-t}{CR}\right)$$

we see that the shape of the curve is defined in each case by the circuit time constant C times R . The greater the product of C and R the longer it will take for a capacitor to charge or discharge.

7.1 In practical terms the time constant can have two interpretations.

a) It is the time taken for the capacitor to charge by 63% of the difference between the initial voltage on the capacitor and the applied voltage V_S .

i.e. If we insert $t = CR$ into the above equations for the voltage on a capacitor we will see it will charge (or discharge) by 63% of the applied voltage in CR seconds.

By how much will it charge (or discharge) in the next CR seconds?

Well, if it has charged (or discharged) by 63% in the first CR seconds in the next CR seconds it will charge (or discharge) by 63% of the remaining 37%. This means that in two time constants the capacitor will charge (or discharge) by 86% of the applied voltage, etc.

- b) In paragraph 6.1 we found that the initial rate of increase in the voltage on the capacitor is given by

$$\frac{dv_C}{dt} = \frac{E}{CR} \quad \text{volts/second}$$

We also noted that this rate of increase falls as the capacitor charges-up. But if this initial rate of increase in V_C were maintained then the time taken for the capacitor to charge fully would be equal to

$$\frac{\text{Applied Voltage}}{\text{rate of change of voltage}} = \frac{E}{\frac{E}{CR}} = CR \text{ secs}$$

which gives rise to our second definition:

The time constant can be defined as "the time taken for the voltage on the capacitor to reach its final value had the initial rate of charge been maintained".

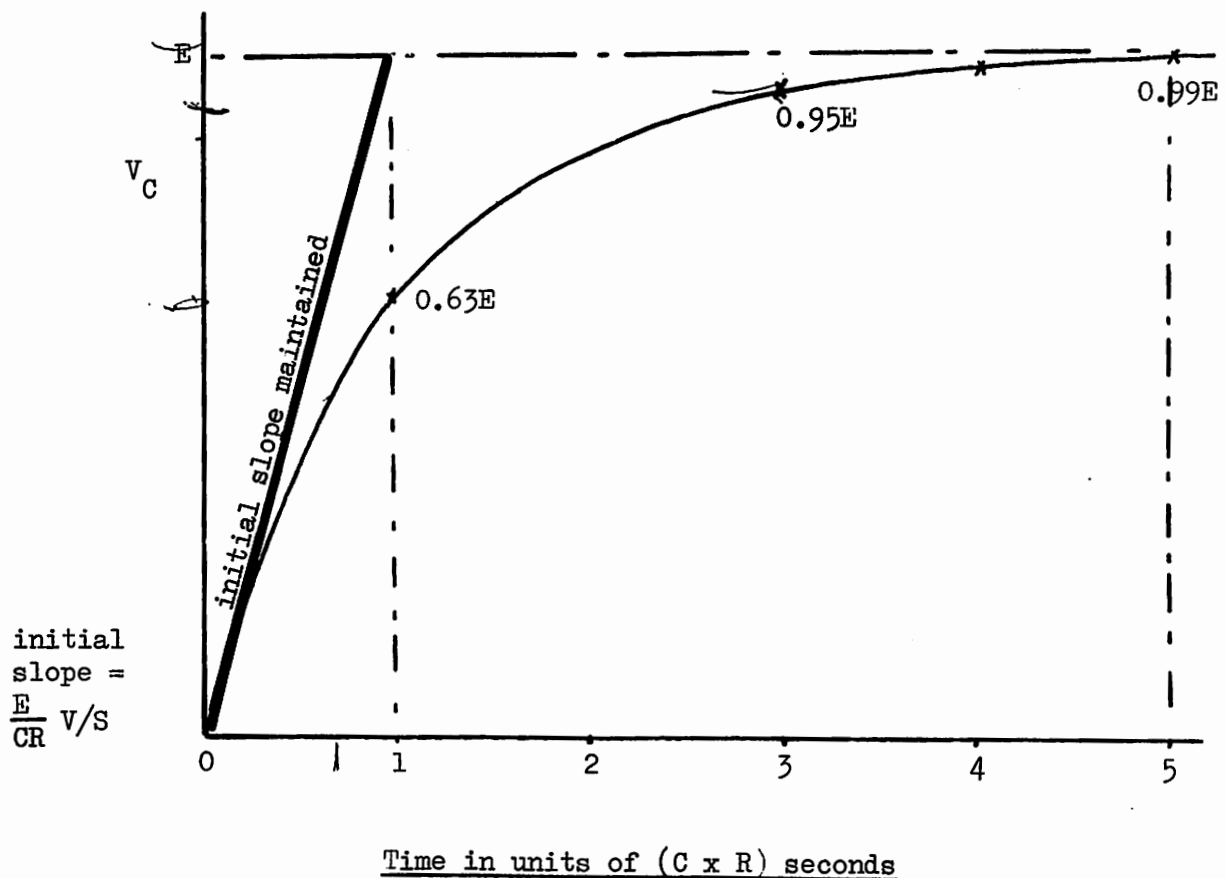


Figure 15 - Charge of a Capacitor

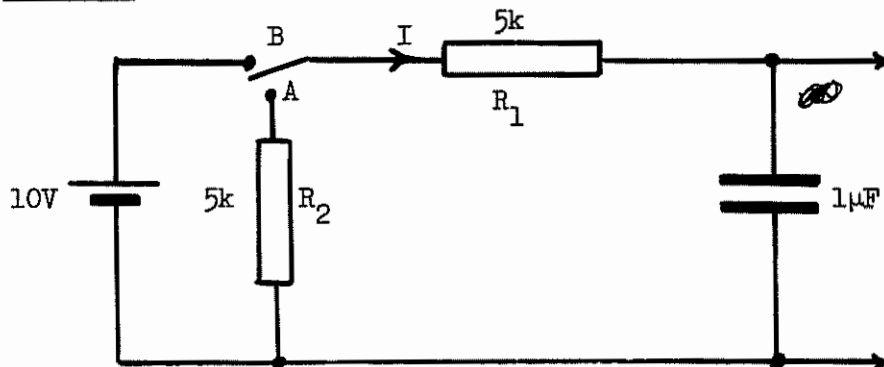
7.2 Time Taken to Charge or Discharge a Capacitor

In a time equal to CR seconds a capacitor will charge or discharge by 63%.

Inserting times equal to $3CR$ and $5CR$ into the formulas in paragraph 7 will show that a capacitor will charge or discharge by approximately 95% and 99.5% respectively in these times.

A capacitor can be taken to be fully charged (or fully discharged) in $5 CR$ seconds

7.3 Exercise



Q1 After the switch has been at position A for some time it is moved to position B.

Determine:

- The value of V_C , V_R and I the instant the switch contact is made.
- The time taken for V_C to reach 6.3V and 9.5V.
- The time taken for V_C to reach 5V.
- The value of V_{R1} after one time constant.
- The time taken for the current to fall to zero.

Q2 When V_C reaches 10V the switch is moved to position A.

Determine:

- The value of V_C , V_{R1} and I the instant the switch makes contact.
- The time taken for the capacitor to discharge.

c) The value of V_C after one time constant and after 3 time constants.

d) The time taken for C to discharge to 2.5V.

$$(a) V_0 = 0 \quad I = \frac{10}{5k} = 2mA$$

$$V_R = 10V$$

$$(b) T = CR$$

$$T_{for 9.5V} = 3CR$$

$$= 15mS$$

$$\therefore \text{time} = 1\mu F \times 5k$$

$$= 5mS$$

$$(c) V_C = V_1 (1 - e^{-t/CR}) = 10. (1 - e^{-t/CR})$$

$$\frac{5}{10} = 1 - e^{-t/CR}$$

$$.5 = e^{-t/CR}$$

$$CR = 5mS$$

$$\log .5 = -\frac{t}{CR} \log_e e$$

$$= -\frac{t}{CR}$$

$$= 3.46mS$$

$$(d) V_R = 3.7V$$

$$(e) I_0 = 25mS$$

$$CR = 10mS$$

$$(b) = 50mS$$

$$(c) V_C = V e^{-\frac{t}{CR}}$$

$$= .5 \text{ Volts}$$

$$(d)$$

$$2.5 = 10 e^{-\frac{t}{CR}}$$

$$\frac{2.5}{10} = e^{-\frac{t}{CR}}$$

$$\ln \frac{2.5}{10} = -\frac{t}{CR}$$

$$= 13mS$$

$$Q2 (a) V_C = 10V \quad I = 1mA$$

$$V_R = 5V$$

ANSWERS

1.a. $V_C = 0$ $V_{R1} = 10V$ $I = 2mA$

b. 5ms and 15ms

c. 3.47ms

d. 3.7V

e. 25ms

2.a. $V_C = 10V$ $V_{R1} = +5V$ $I = -1mA$

b. 50ms

c. 3.7V and 0.5V

d. 13.9ms

8. CR CIRCUITS AND REPETITIVE RECTANGULAR VOLTAGE WAVEFORMS

The effect of a series CR circuit on a rectangular waveform depends on:

- a. The relationship between the time constant of the circuit and the pulse duration of the applied waveform.
- b. Whether the output is taken across the capacitor or resistor.

8.1 Short CR circuits

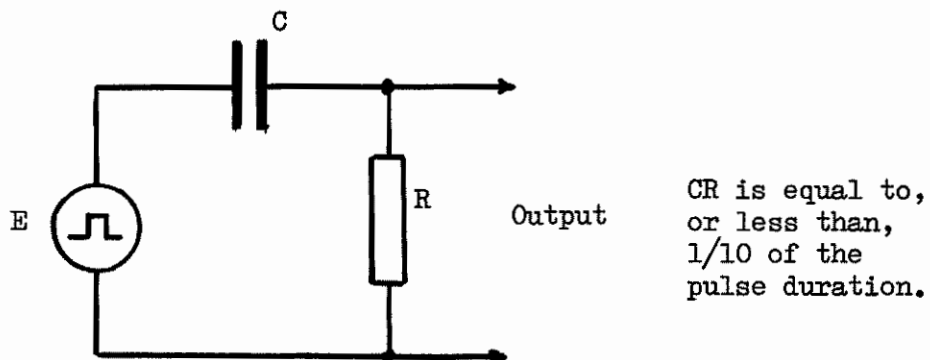


Figure 16 - Short CR

A 'short CR' circuit is one in which the capacitor is able to charge or discharge well within the positive and negative going pulse durations of the input waveform.

A short CR is where

$$CR \leq 1/10 \text{ pulse duration } (t) \text{ of the input waveform.}$$

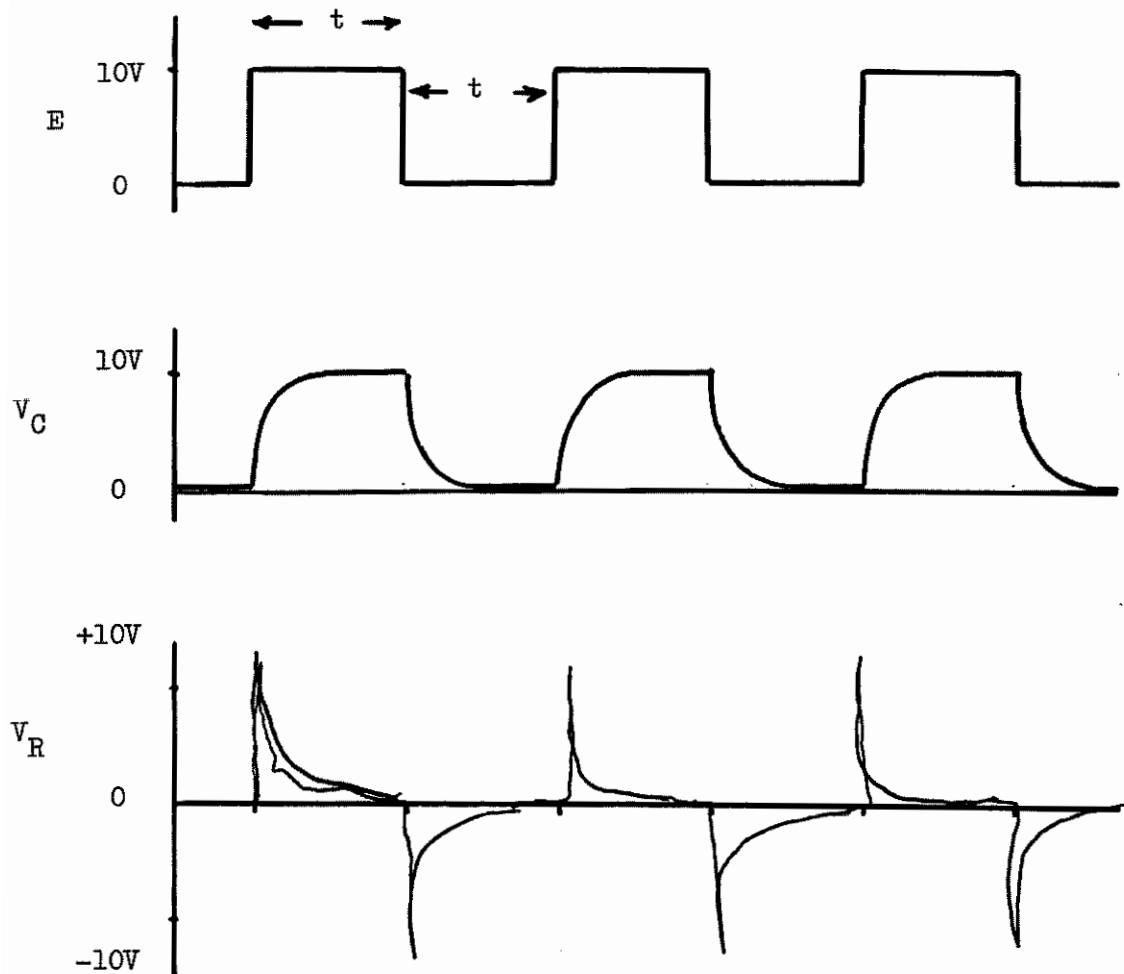


Figure 17 - Waveforms associated with a short CR circuit

Consider the waveforms in figure 17. If we let

$$CR = \frac{t}{10}$$

The capacitor can be taken to completely charge or discharge in a time equal to $5CR$ seconds

$$\text{i.e. } 5CR = 5 \times \frac{t}{10} = \frac{t}{2}$$

This fact enables us to deduce the waveform for V_C as shown in figure 17.

The waveform across V_R will be equal to

$$V_R = E - V_C$$

8.2 Subtract the waveform V_C in figure 17 from E and sketch in the waveform for V_R .

8.3 Practical

A CR circuit is required that will act as a short CR to a 1kHz 10V squarewave. If the capacitor has a value of 10nF, determine a suitable value for the resistor.

8.4 Build this circuit on a breadboard. Obtain the input waveform from a signal generator. Display the input and the V_R waveform on an oscilloscope and check that you have drawn the waveform for V_R in figure 17 correctly.

8.5 Measure the peak to peak amplitude across V_R . It should be $\pm 10V$. If your waveform is not this amplitude it may be due to the internal resistance of the waveform generator. Why should this affect the amplitude of the waveform across the resistor?

Well, the waveform ' V_R ' will appear across the total resistance in series with the capacitor of which the generator internal resistance will be part. The output waveform will therefore tend to be slightly smaller in amplitude than 20V p-p depending on the value of this internal resistance of the signal generator.

9.

DIFFERENTIATION

You will remember from the maths package that differentiating a rectangular waveform should produce a series of positive and negative spikes. This is shown in figure 18.

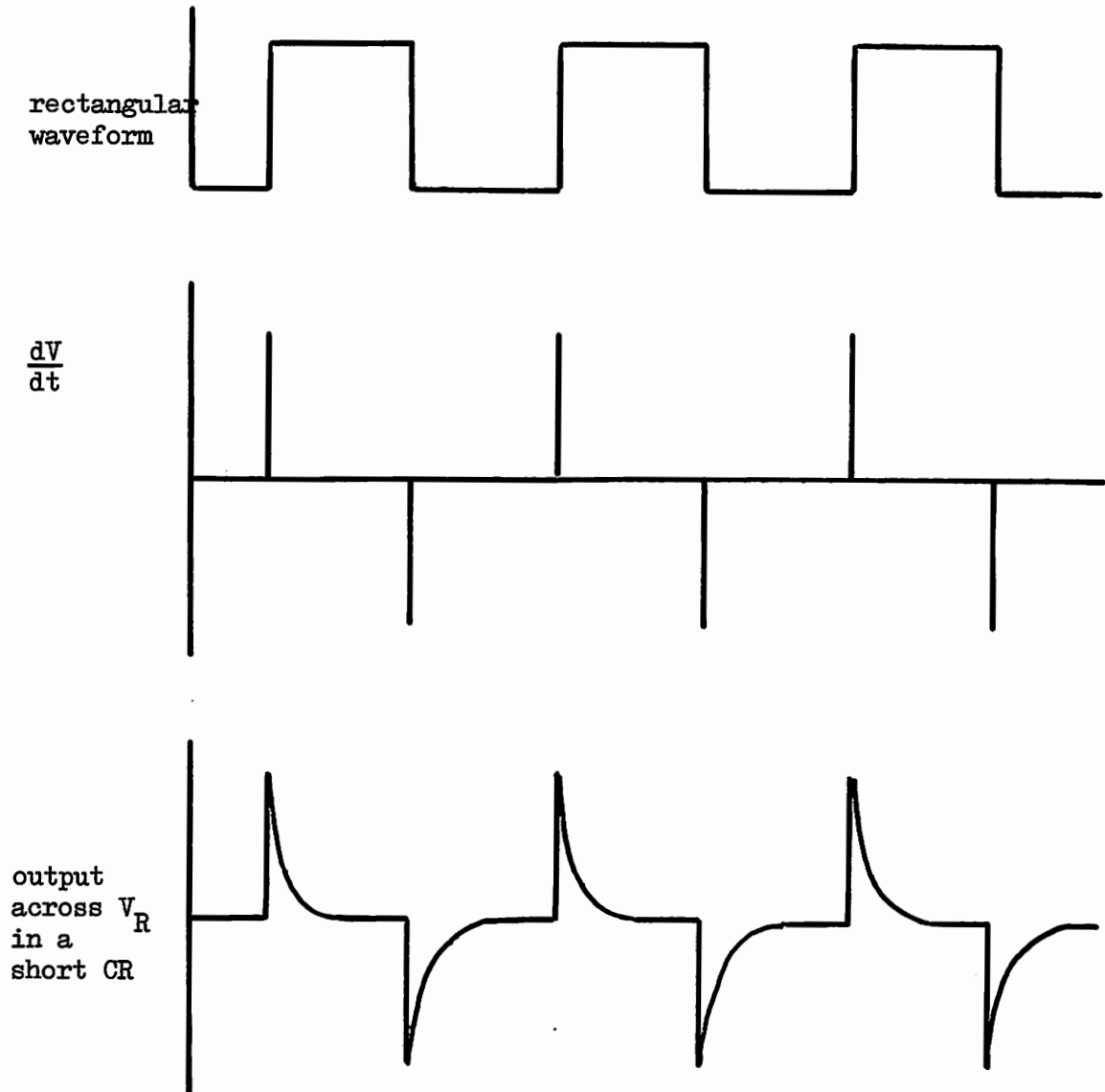
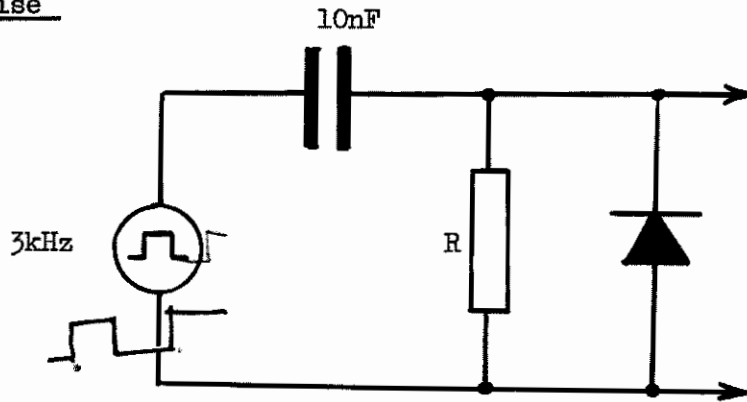


Figure 18 - Differentiation

The output waveform across V_R in a short CR approximates to $\frac{dV}{dt}$. The shorter the CR the better this approximation becomes. Hence a short CR with the output taken across R is often referred to as

A Differentiating Circuit

9.10 Exercise



- Determine a suitable value for R if the CR is to differentiate the input waveform.
- Sketch the output waveform from the above circuit. Assume the diode to have infinite resistance when cut-off and negligible resistance when conducting.
- What practical use could be made of a circuit such as the one shown above.

$$CR = \frac{1}{F \cdot 10} = \frac{1}{10 \cdot 10^9 \cdot 3 \cdot 10^3} = 10^{-13}$$

$$CR = \frac{1}{10F}$$

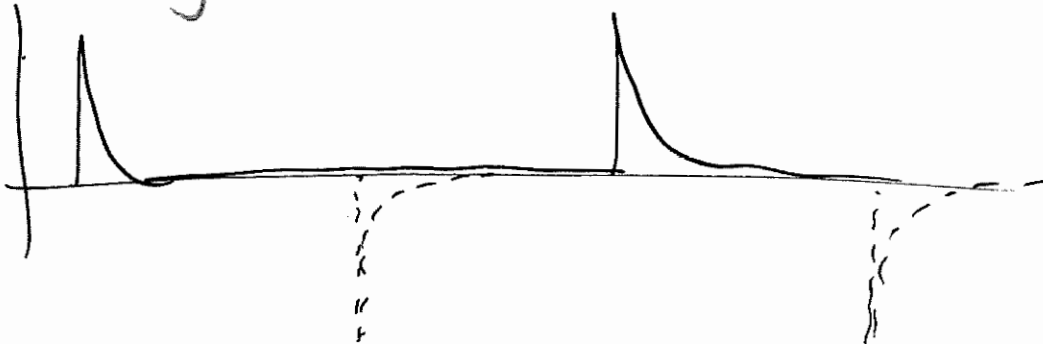
$$R = \frac{1}{10F \cdot C}$$

$$\frac{3 \cdot 3K}{2}$$

$$\approx 1.6K$$

$$T = \frac{1}{2f}$$

(b)



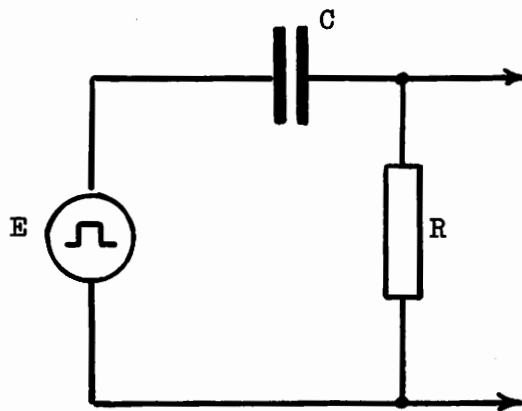
(c) Gated trigger Pulse !!

Answers

- a. Less than $1.6k\Omega$
- b. A series of narrow positive going pulses.
- c. To generate a series of trigger pulses.

10.

LONG CR CIRCUITS



CR is equal to, or greater than, 10 times the pulse duration.

Figure 19 - Long CR Coupling Circuit

In a 'long CR' circuit, the time constant of the circuit is very long compared to the pulse duration of the applied waveform. The capacitor charges and discharges by only a small fraction of the applied voltage during each input pulse.

In a long CR

$$CR \geq 10 \text{ times the pulse duration (t)}$$

- 10.1 The circuit action in a long CR can be split into two periods. The period immediately following the application of the waveform, a time during which the capacitor generally charges up more than it discharges on each half of the input waveform. After a few seconds the steady-state conditions are reached. The capacitor then charges as much on positive going input pulses (of the input waveform) as it discharges on the negative going pulses.

10.2 Initial Conditions

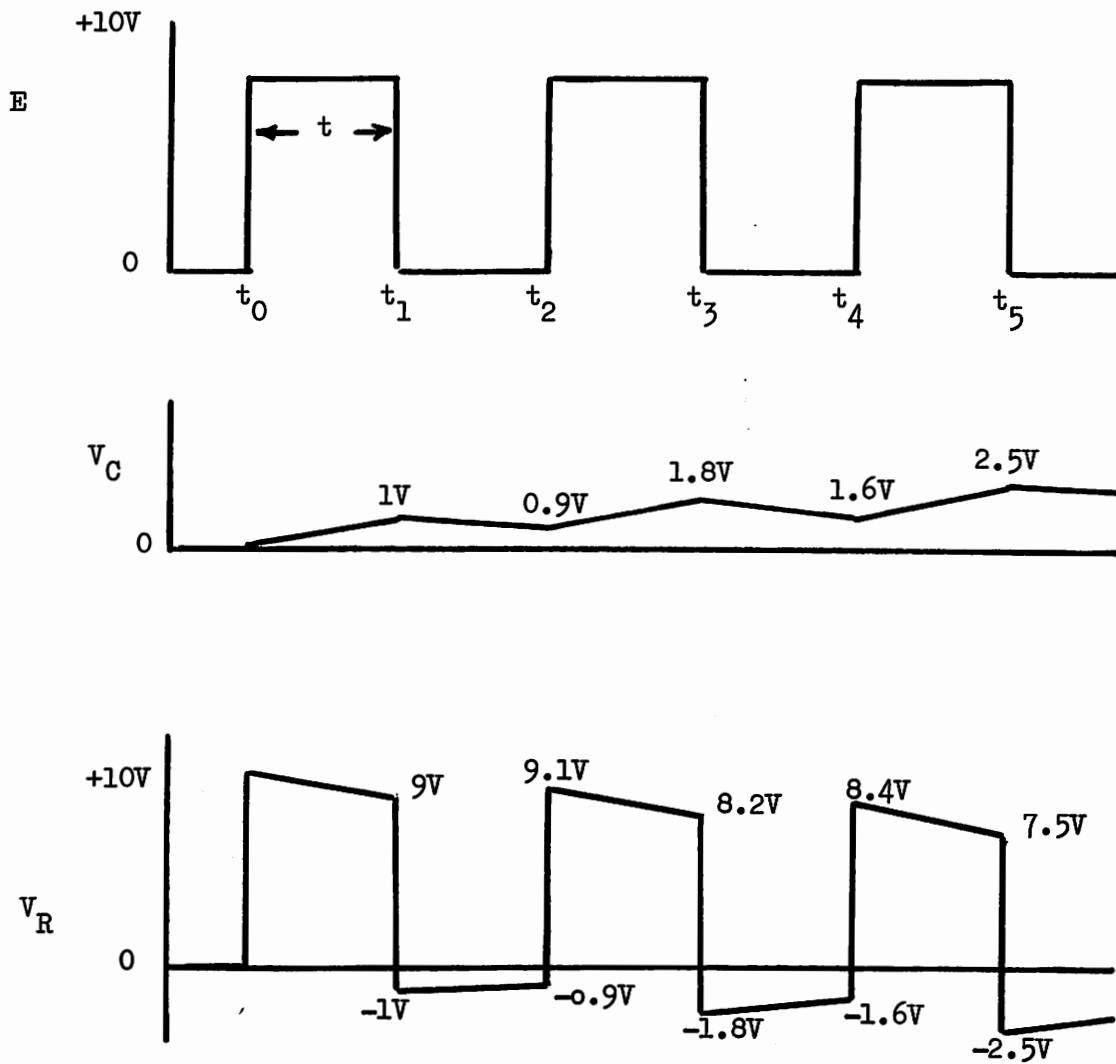


Figure 20 - Long CR initial conditions

Consider the initial conditions in the circuit of figure 19. The rate of increase in the voltage on the capacitor when each pulse is applied can be calculated from (From paragraph 6.1)

$$\frac{dV_C}{dt} = \frac{E - V_C}{CR}$$

Because the circuit is a long CR, the capacitor will charge for a period of time t , where $t = \frac{CR}{10}$

hence the voltage on the capacitor will rise by

$$\frac{dV_C}{dt} \times t = \frac{E - V_C}{CR} \times \frac{CR}{10} = \frac{E - V_C}{10}$$

i.e. $1/10$ of the difference in potential between the voltage on the capacitor and the applied voltage.

Hence:

- a) during the period to t_1 V_C is approximately zero hence as $E = 10V$ V_C will rise by $1/10$ of $10V$, i.e. one volt.
 V_R will equal $E - V_C$, and hence must fall by one volt, see figure 20.
- b) between t_1 and t_2 V_S falls to zero hence V_C will rise $1/10$ of $(0V - 1V)$ i.e. fall $1/10V$, falling to $0.9V$.
- c) at time t_2 E again rises to $10V$ but with V_C at $0.9V$ V_C will rise $1/10$ of $(10V - 0.9V)$ hence at t_3 V_C will have risen to $1.81V$ and V_R will have fallen by $1.81V$ to $8.19V$.

10.3 This process will repeat each pulse, the figures are given on figure 21. The following points should be noted.

- a) The mean voltage on the capacitor is rising toward the mean level of the input waveform.
- b) On positive going pulses the increase in V_C is slightly smaller pulse by pulse, whilst each decrease in V_C , on negative going pulses, progressively increases. A balance in V_C will occur when the increase on positive going pulses equals the decrease during negative going pulses.

10.4 Steady State Conditions

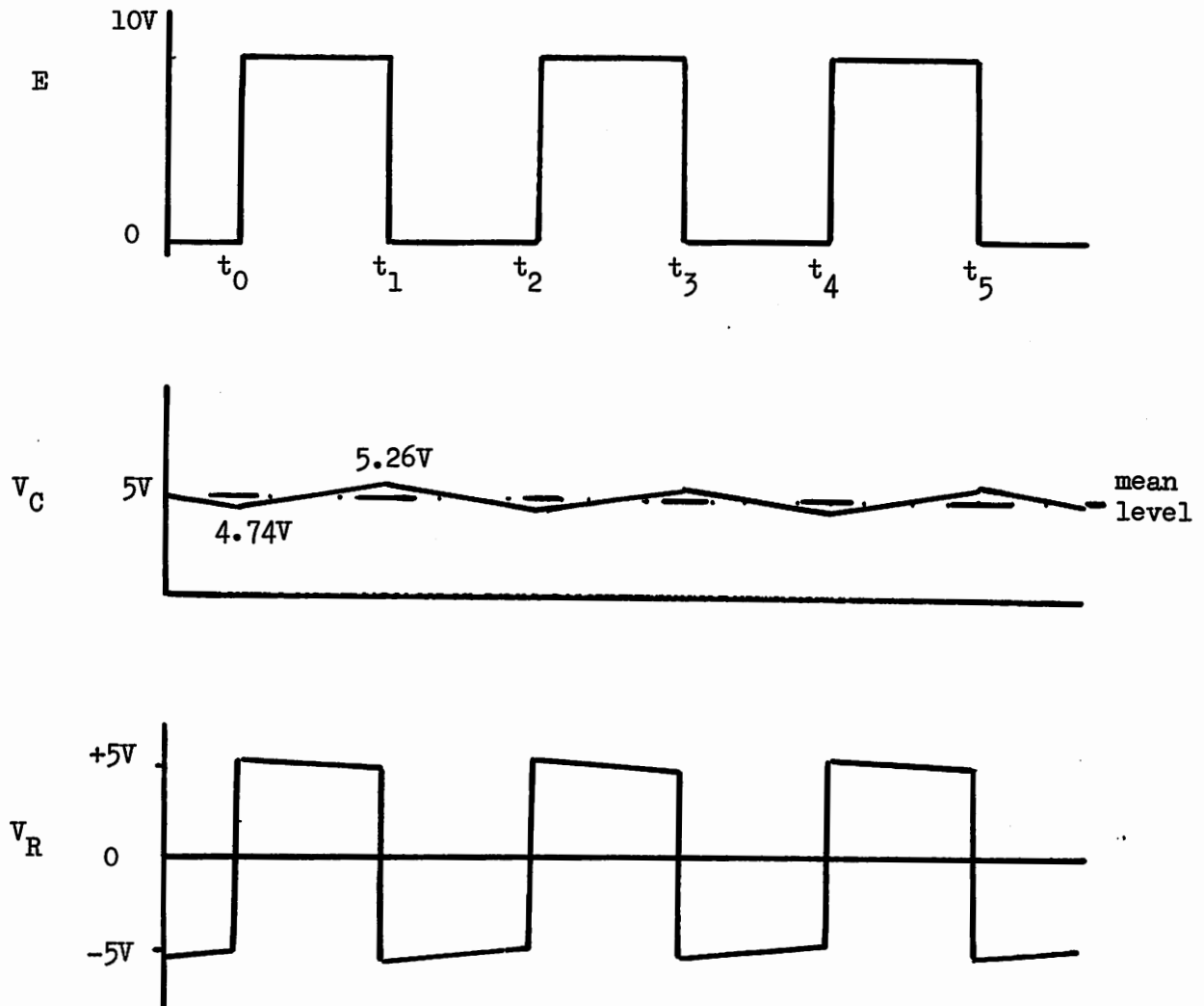


Figure 21 - Long CR - Steady State Conditions

In figure 21 we see the situation once the steady state conditions have been reached.

At t_0 $V_C = 4.74V$ hence when the input rises to +10V the capacitor charges during the pulse by

$$1/10 \text{ of } (10 - 4.74) = 0.526V$$

hence at t_1 $V_C = 4.74 + 0.526V = 5.26V$

When the input falls to 0V the capacitor will discharge by:

$$1/10 \text{ of } 5.26V = 0.526V$$

hence at t_2 $V_C = 5.26V - 0.526V = 4.74V$

i.e. the charge and discharge over one cycle of the input waveform are equal.

10.5 The following points should be emphasised:

- a) The mean level of the voltage waveform across the resistor is zero.
i.e. The DC level of the input waveform has been removed.
(It appears across C).
- b) The shape of waveform across the resistor approximates to that at the input but without its DC level. The longer the CR the less of an approximation this will be.
The property of this circuit to block d.c. whilst passing the a.c. waveform undistorted has led to its extensive use as a coupling circuit in multistage amplifiers.
- c) The term used to describe the fall in the voltage at the output of a long CR coupling circuit is 'sag'.

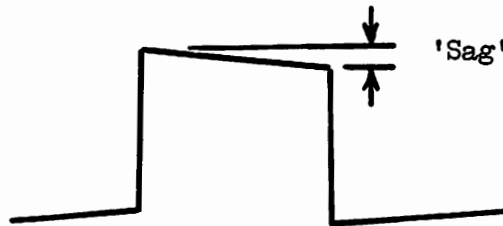


Figure 22 - Sag

The longer the CR the less will be the 'sag'.

11.

INTEGRATION

The maths package showed that the effect of integrating a square waveform is to produce a triangular shaped waveform approximating to the voltage waveform across a capacitor in a Long CR. This is illustrated in figure 23.

CR Long to the input waveform

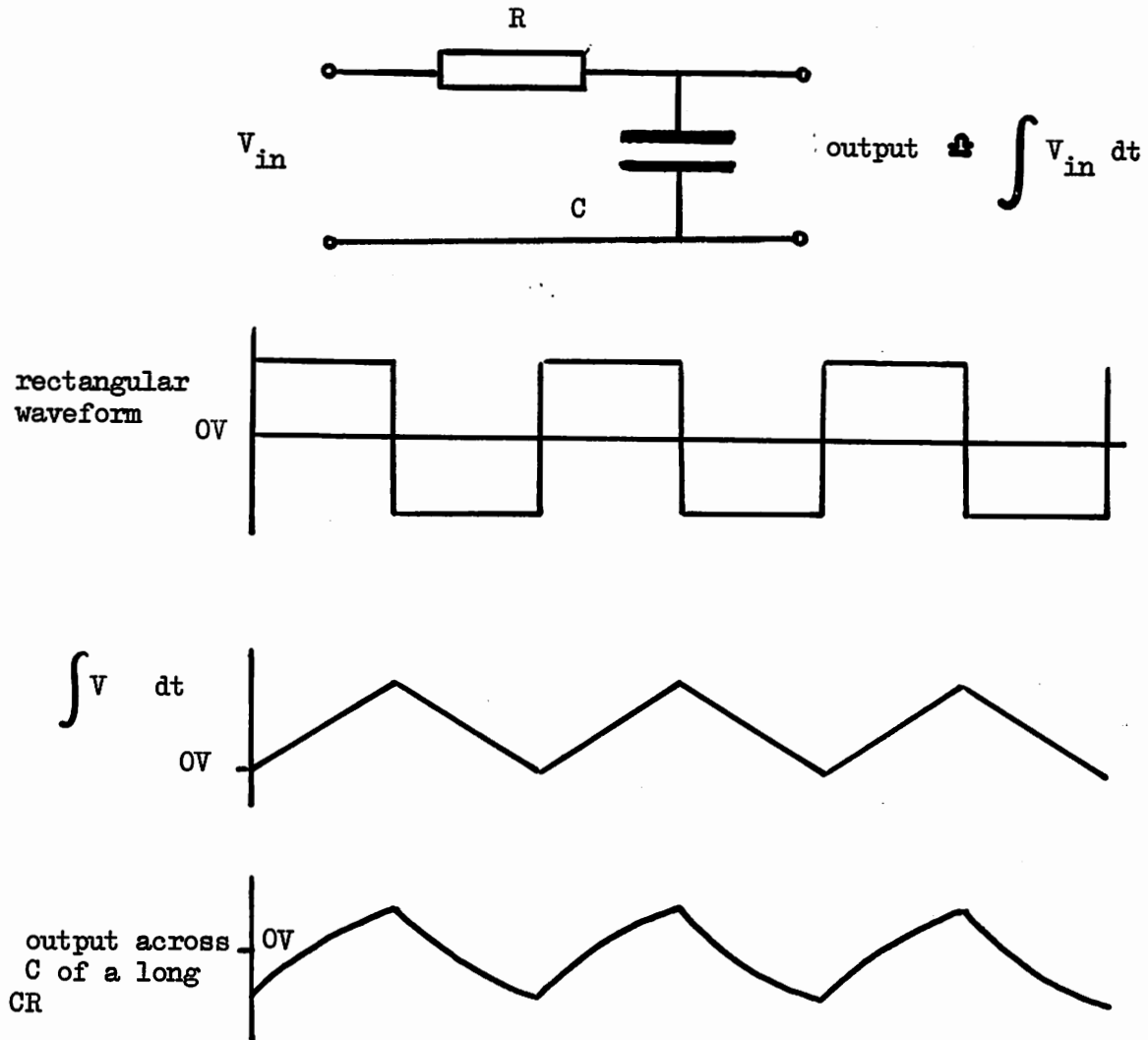


Figure 23 - Long CR integrating circuit

A long CR in which the output is taken across the capacitor is often referred to as

an Integrating Circuit

11.1 Practical

Determine a suitable value of resistor that will act with a $0.1\mu\text{F}$ capacitor to produce:

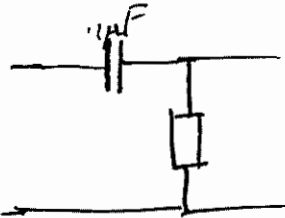
1. a CR coupling circuit
2. an integrating circuit

for a 10V p/p 1kHz square wave.

11.2 Build these circuits on a breadboard and apply a suitable input waveform from the signal generator. Observe the input and output waveforms on an oscilloscope.

Note the effect of increasing and reducing the frequency of the input waveform to each circuit. Record your findings below.

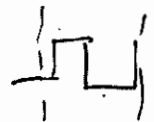
1/ 1kHz $P_{dur.} = 5 \cdot 10^{-3}$



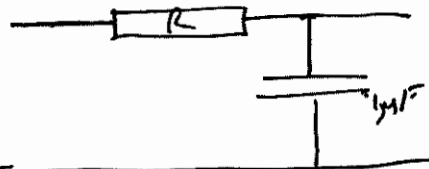
$$CR \gg 10P_d$$

$$CR = 5\text{ms}$$

$$R = \frac{5\text{ms}}{0.1\mu\text{F}} \\ = 5\text{k}\Omega$$



2/

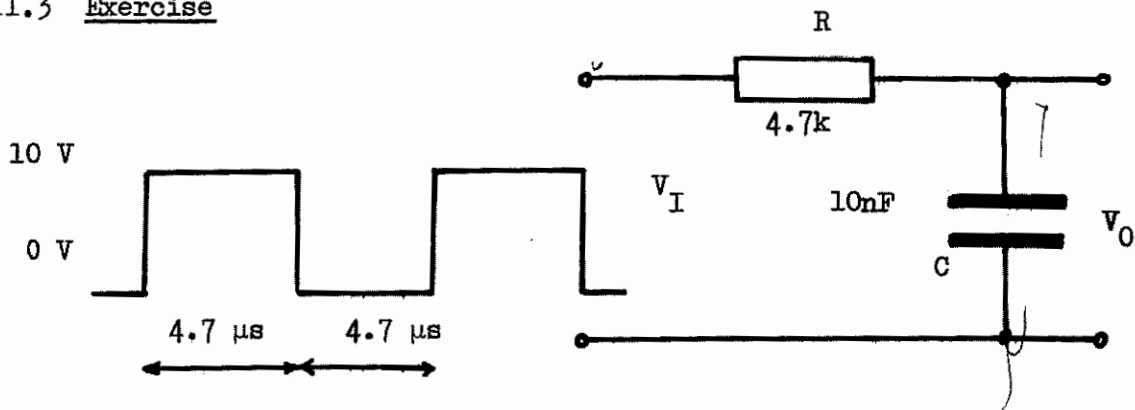


$$CR \gg 10 \cdot \text{Pulse duration}$$

$$CR = 5\text{ms}$$

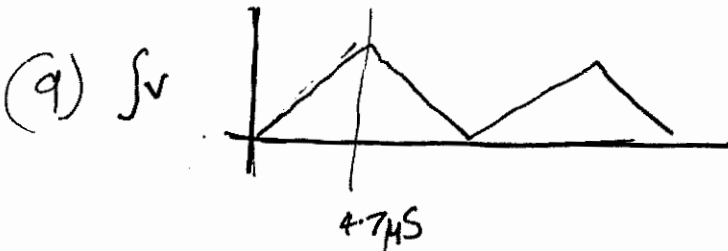
$$R = 5\text{k}\Omega$$

11.3 Exercise



The square wave shown above is applied to the CR network shown.

- Will the output of this circuit be a reasonable approximation to the integral of the input waveform. Briefly explain your reasons.
- Sketch the output voltage waveform including the mean DC level and calculate the approximate peak to peak output signal.

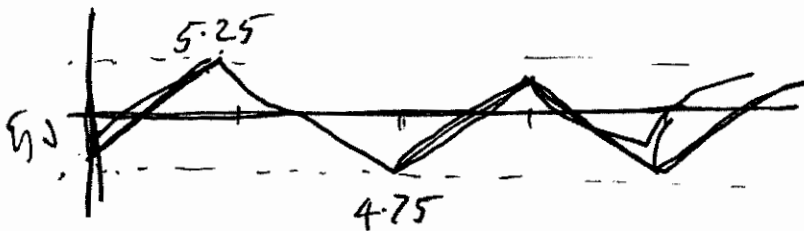


$$CR = 10 \text{ nF} \cdot 4.7 \text{ k}\Omega = 47 \mu\text{s}$$

$$\text{Pulse duration} = 4.7 \mu\text{s}$$

$\therefore CR \gg \text{Pulse duration}$ hence a good approximation.

(b)



$$\Delta Q = 1 \mu C$$

$$Q = C \Delta V$$

$$I_R = \frac{10}{2 \cdot 4.7k}$$

$$\Delta Q = \frac{4.7 \mu S \cdot 5}{4.7k} = \frac{4.7 \times 10^{-6} \cdot 5}{4.7 \times 10^3}$$

$$= 5 \times 10^{-9}$$

$$=$$

$$Q = C \Delta V$$

$$5 \times 10^{-9} = C \Delta V$$

$$\frac{5 \times 10^{-9}}{10 \times 10^{-9}} = \Delta V$$

$$\underline{\underline{\Delta V = 0.5 V}}$$

$$V_S = V_R + V_C$$

$$V_C = V_S + V_R$$

$$V_C = V_S / 2$$

$$V_R = V_S / 2$$

$$V_R = I R$$

$$I = V_S / R$$

ANSWERS to paragraph 11.3.

The Time Constant $CR = 10 \times 10^{-9} \times 4.7 \times 10^3$
 $= 47 \mu s$

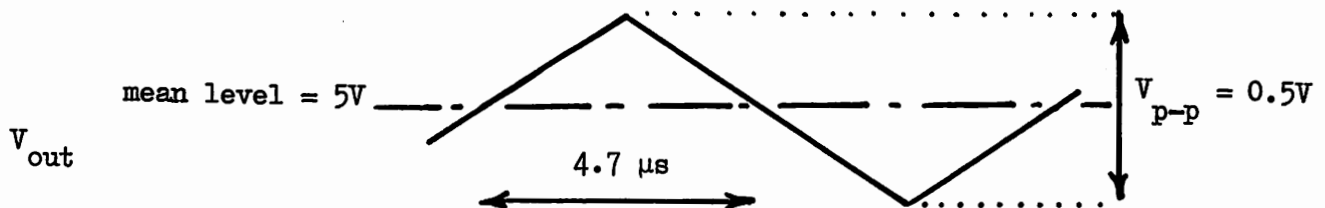
The Pulse Duration $(t) = 4.7 \mu s$

So, $CR = 10 t$

This indicates that the output will be a good integral of the input signal (viz. a "triangular wave"). When the circuit has settled down to the steady state condition the mean DC level of the output will equal the mean DC level of the input. In this particular case the input signal has a 1 : 1 mark to space ratio.

So, the mean DC level of the input $= \frac{1}{2} V_S$ where $V_S = 10V$
 $= V_S/2$

Therefore mean DC level of the output $= V_S/2$
 $= 10/2$
 $= 5 \text{ Volts}$



The amplitude of the triangular output waveform can be found by determining the amount the capacitor will charge and discharge during each input cycle.

When the input voltage (V_S) rises to +10V the voltage across the resistor will be

$$V_R = V_S - V_C$$

as V_C is charged to $\frac{V_S}{2}$,

$$V_R = \frac{V_S}{2}$$

The current is given by

$$I = \frac{V_R}{R}$$

$$I = \frac{V_S}{2R}$$

Now, we know that $\frac{dV_C}{dt} = \frac{I}{C}$

Thus, $\frac{dV_C}{dt} = \frac{V_S}{2CR}$ Volts/sec

Because the charging and discharging times are very short compared to the time constant this charging rate does not change significantly.

The capacitor voltage therefore rises linearly:

The peak to peak output voltage = $\frac{V_S}{2CR} \times t$ ($t = \text{pulse duration}$)

So, " " " " = $\frac{10}{2 \times 47 \times 10^{-6}} \times 4.7 \times 10^{-6}$
 = 0.5 Volts (p/p) (5% of V_I)

12.

MEDIUM CR CIRCUITS

This is the situation in which the time constant lies between that considered long or short.

i.e.

$$\frac{1}{10}t < CR < 10t$$

if we consider a circuit in which

$$CR = t$$

Using an argument similar to that used for the long CR the following waveforms would be obtained:

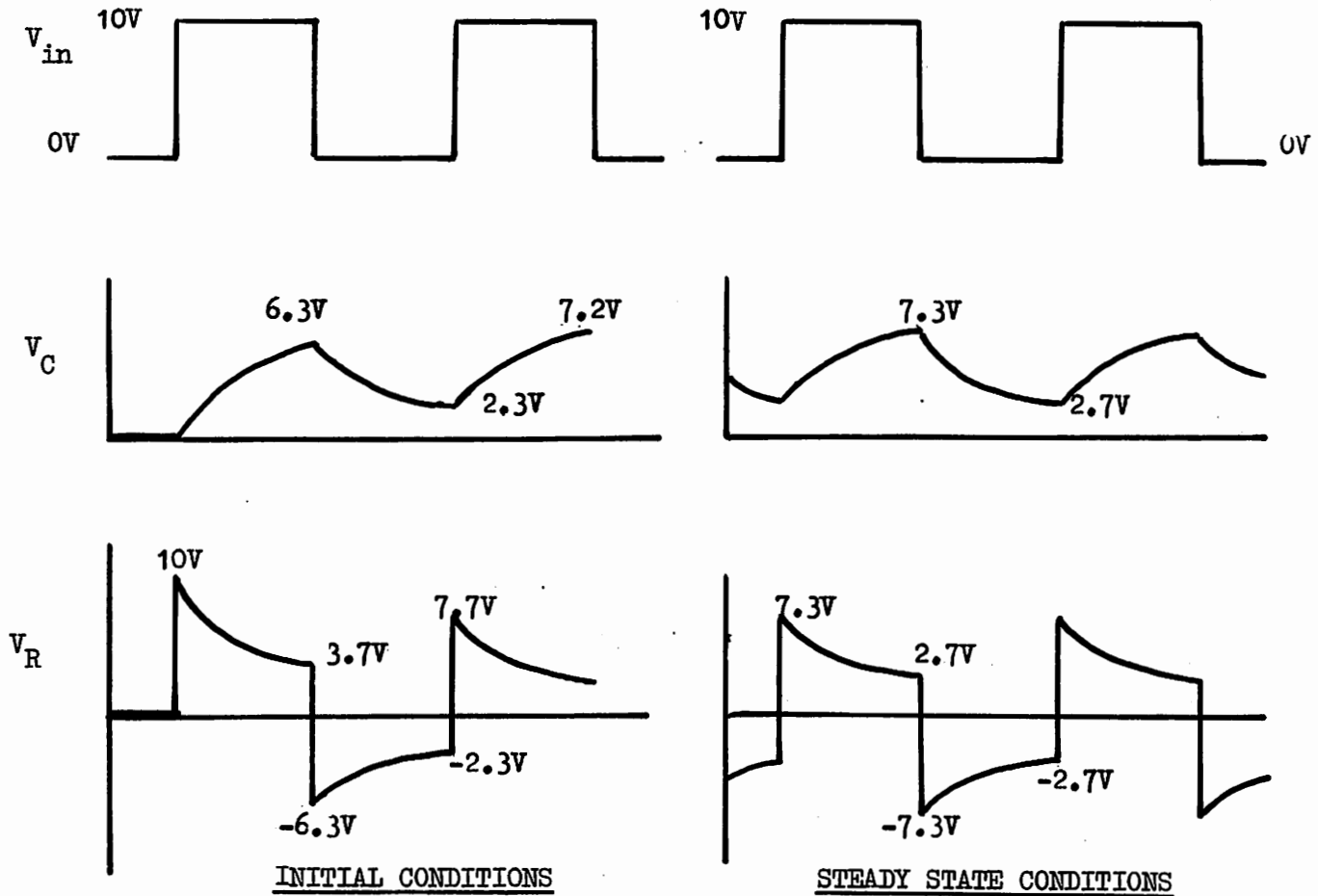
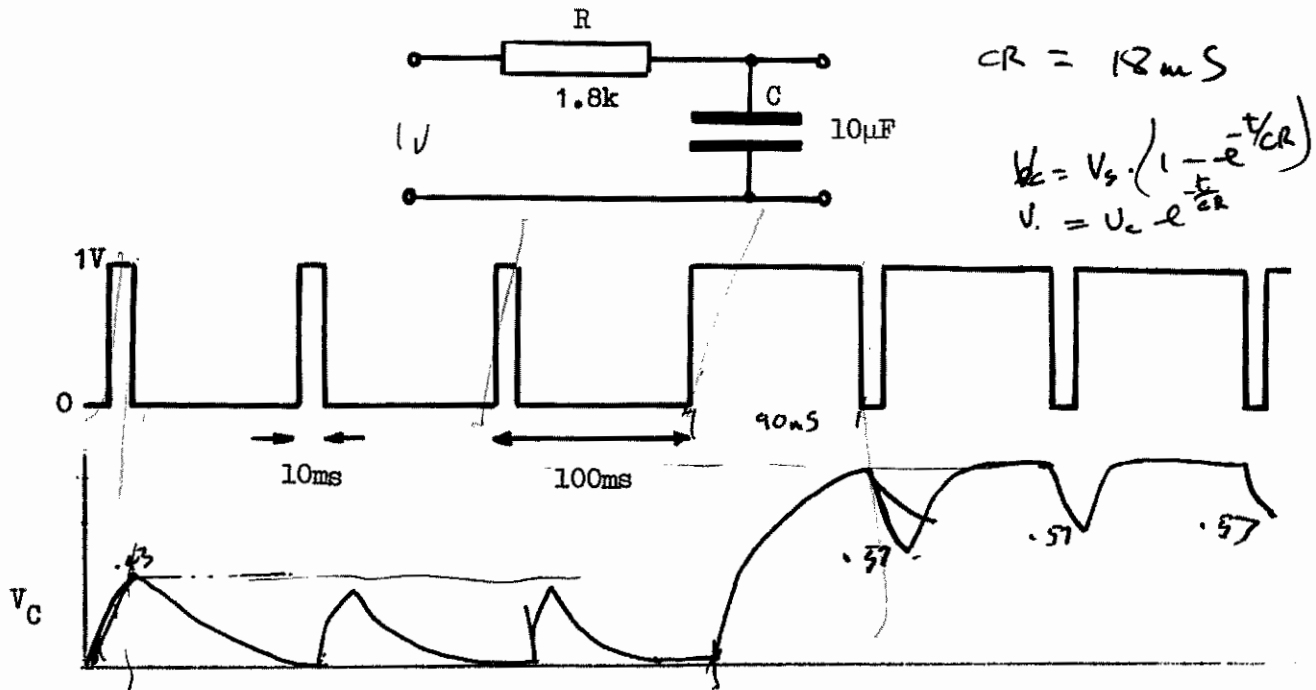


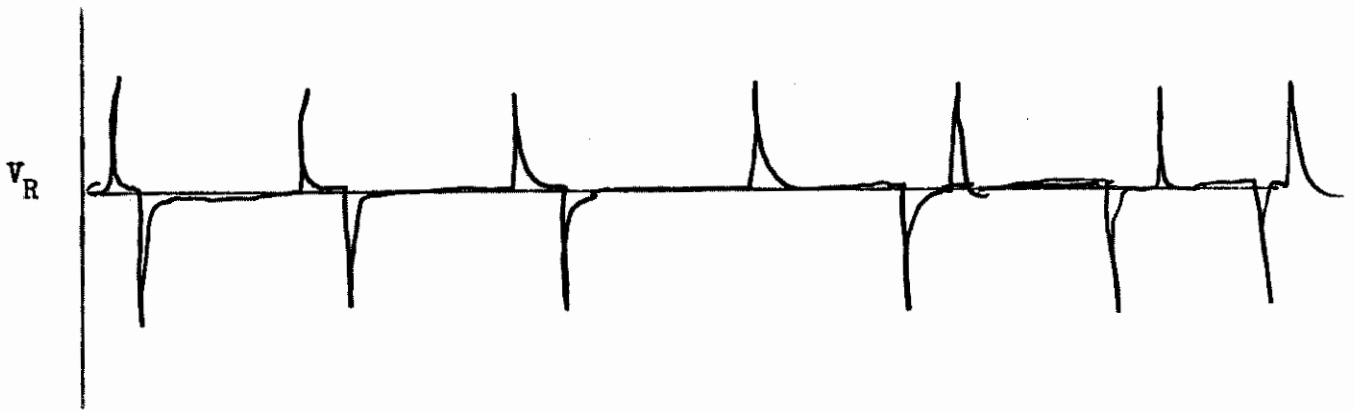
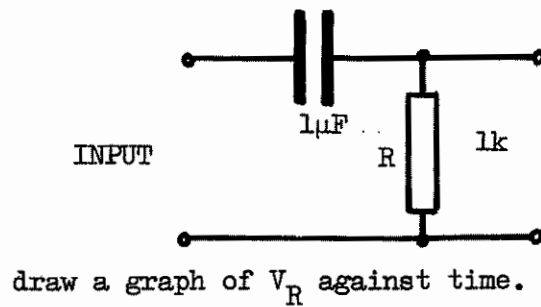
Figure 24: Medium CR - waveforms

12.1. EXERCISE

1. The following waveform is applied to the circuit shown below:-

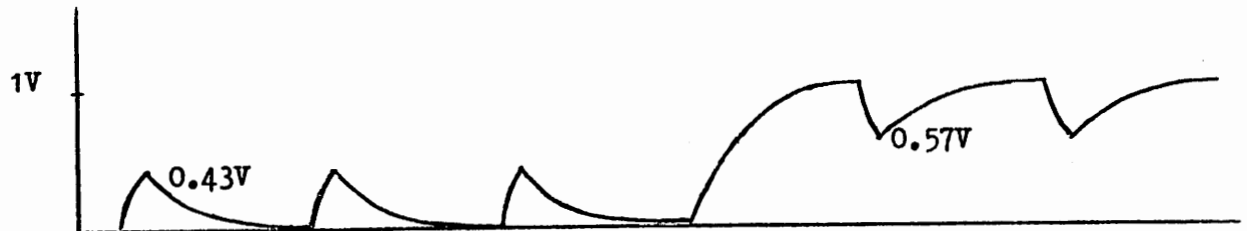


- Sketch under the above waveform a graph of V_c against time.
- The same waveform is applied to the following circuit.



Answer to paragraph 12.1

1.



Circuit time constant = 18ms

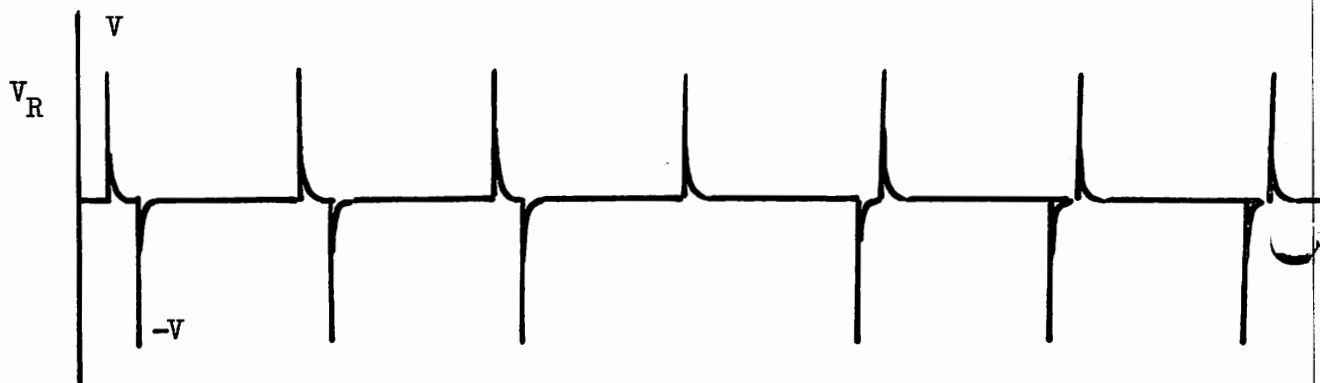
hence during 10ms pulse V_C will change by $1(1 - \exp^{-\frac{10\text{ms}}{18\text{ms}}})$ volts
= 0.426V

Time taken for the capacitor to fully charge or discharge

$$= 5CR$$

$$= 90\text{ms}$$

2.



Circuit time constant = 1ms

Time taken by capacitor to fully charge or discharge

$$= 5\text{ms}$$

13.

A VOLTAGE STEP APPLIED TO A SERIES LR CIRCUIT

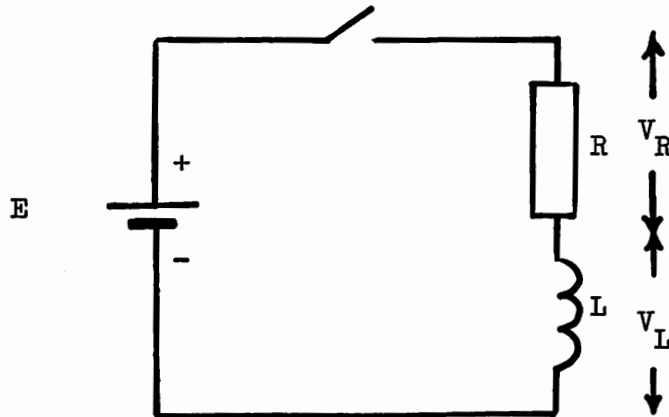


Figure 25: A Series LR circuit

Consider the above circuit at the instant the switch is closed. The back emf generated across the inductor will oppose current flow hence at the instant of switch-on no current flows:

V_L will equal E ,

V_R will be zero and as $I = \frac{V_R}{R}$

I will be zero.

Now the back emf $V_L = L \frac{dI}{dt}$ hence the current will increase initially from zero at a rate $\frac{dI}{dt} = \frac{V_L}{L}$ where $V_L = E$.

hence $\frac{dI}{dt} = \frac{E}{L}$ A/s

as the current rises V_R will rise ($V_R = IR$) and V_L will fall and so will the rate of increase of I .

Using similar arguments for the current in a series LR circuit as was used for the voltage V_C in a series CR circuit we would find that the current rises exponentially to a maximum value determined by the value of R and the applied voltage E .

$$I = I_{MAX} \left(1 - \exp - \frac{tR}{L} \right) \quad \text{where } I_{max} = \frac{E}{R}$$

hence

$$I = \frac{E}{R} \left(1 - \exp - \frac{tR}{L} \right)$$

This is illustrated in Figure 26.

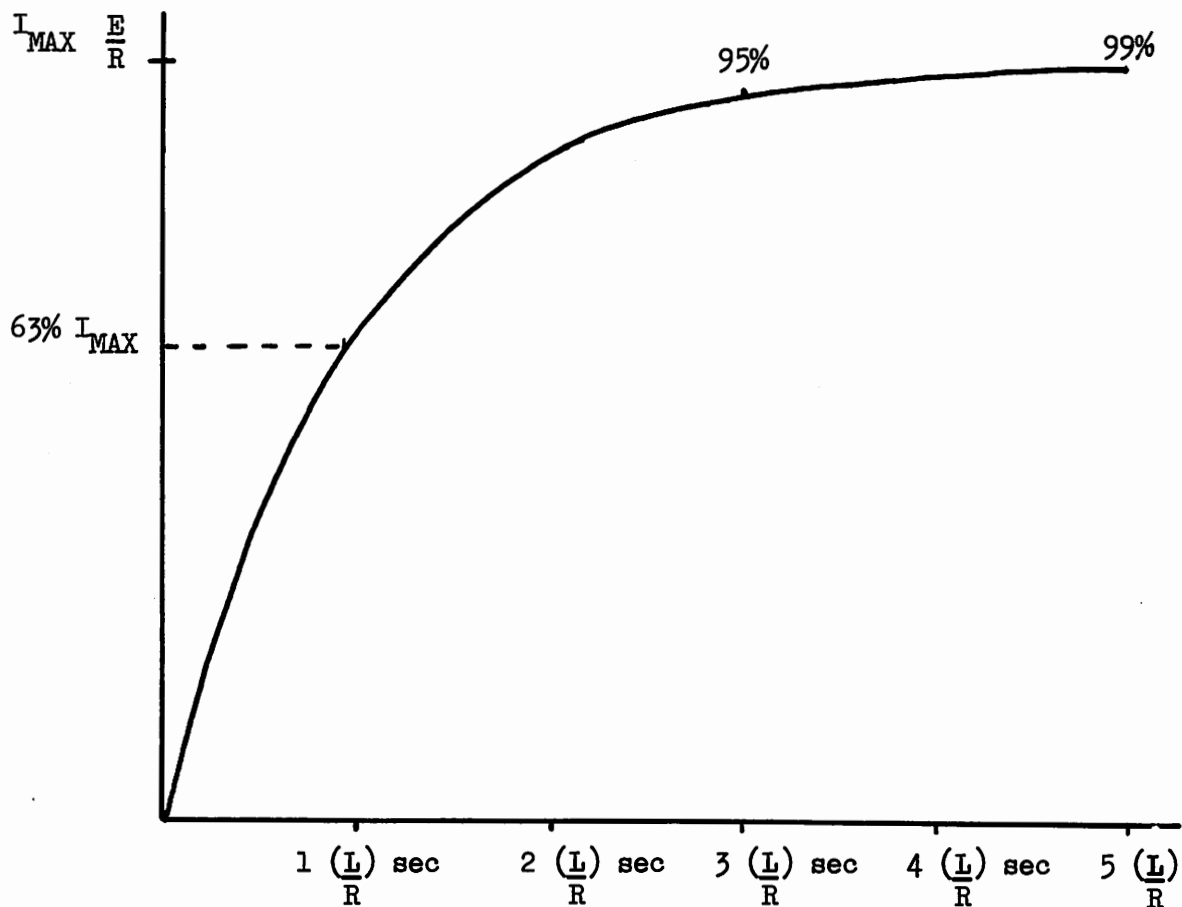


Figure 26: Current in a series LR circuit

13.1 The equation for I at any time (t) can be obtained in a similar manner to the equation for V_C in a CR circuit.

With reference to the circuit of figure 25

$$E = V_R + V_L \quad \text{where } V_R = I \times R$$

$$\text{hence } E = IR + V_L$$

$$\text{now } V_L = L \frac{dI}{dt} \quad \text{hence}$$

$$E = IR + L \frac{dI}{dt}$$

This equation can be solved for I .

If we assume that $I = 0$ at time $t = 0$

$$\text{then } I = \frac{E}{R} \left(1 - \exp \frac{-tR}{L} \right)$$

13.2 By considering the relationship between V_R and I and between E , V_L and V_R use the equations given in paragraph 13.1 to obtain expressions for:

$$V_R =$$

$$V_L =$$

13.3 Consider the circuit of figure 27.

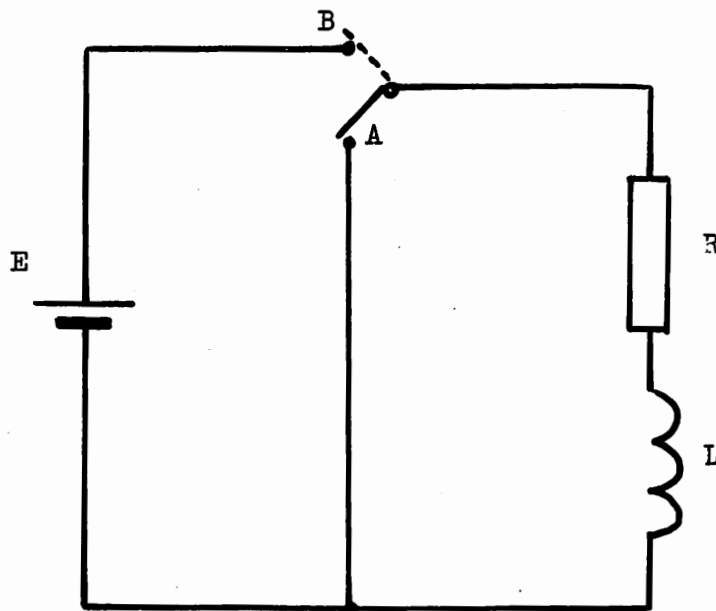
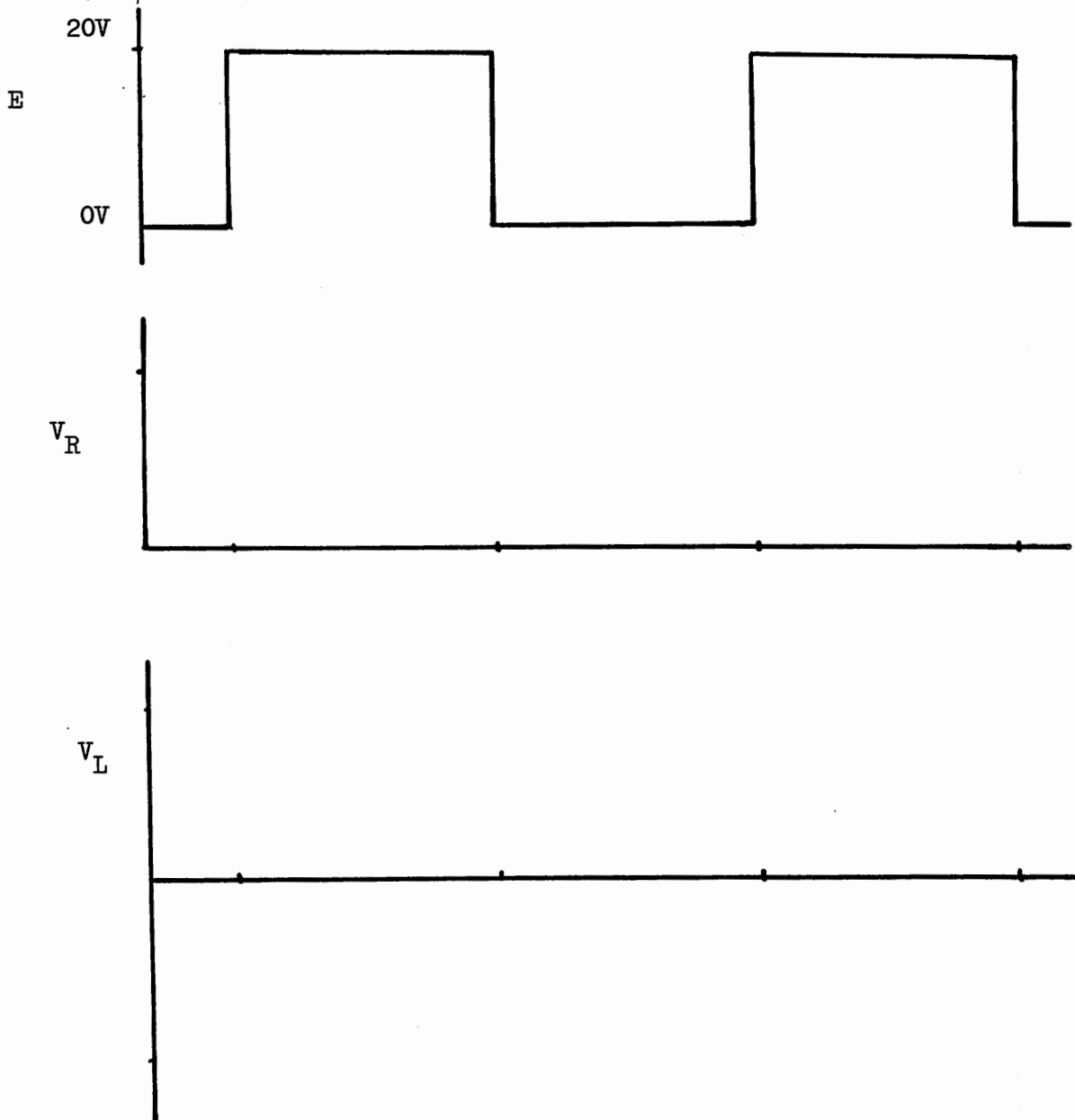


Figure 27 - Series LR Circuit

If the switch is moved between contacts A and B it will apply alternatively 0V and +20V to the LR circuit. This is illustrated in figure 28. Sketch in the corresponding V_R and V_L waveforms assuming the current is able to rise to max or fall to zero before the switch position is altered.

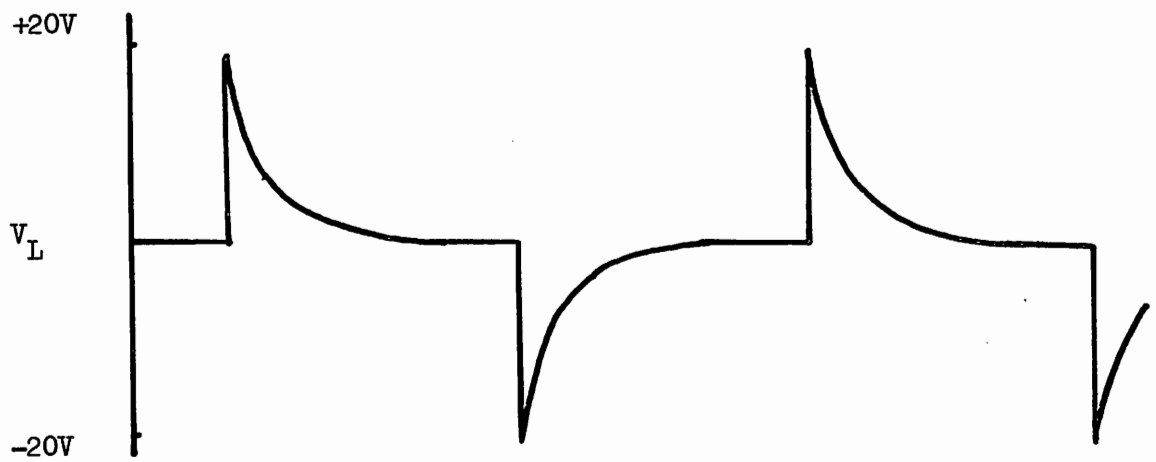
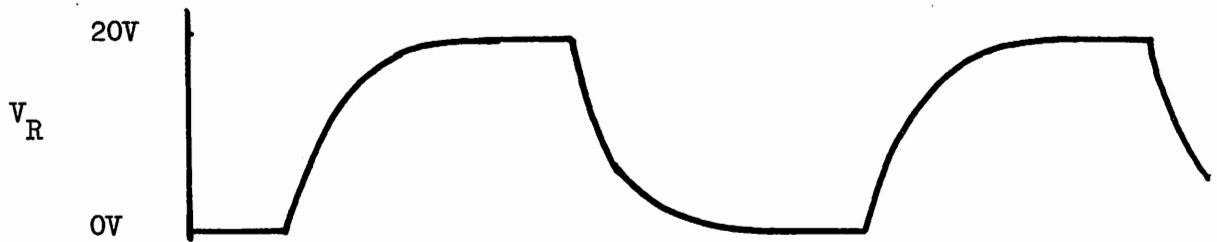
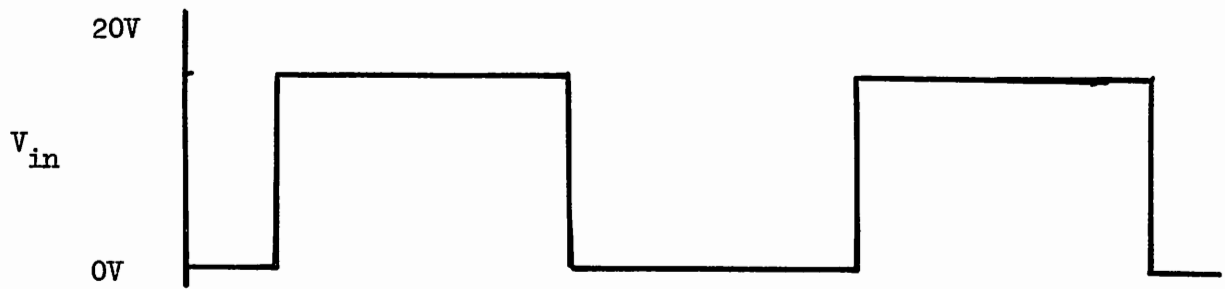


Answers to paragraph 13.2

$$a. \quad V_R = IR = E \left(1 - e^{-\frac{tR}{L}} \right)$$

$$b. \quad V_L = E - V_R = E e^{-\frac{tR}{L}}$$

Answer to paragraph 13.3.



14.

TIME CONSTANT OF AN LR CIRCUIT

If we consider the equations for I , V_L and V_R in a series LR circuit, we see that the shape of the curve is defined by $\frac{L}{R}$

hence

$\text{Time constant} = \frac{L}{R} \text{ seconds}$
--

The greater the inductance or smaller the resistance in the circuit the longer it will take for the current to rise to its maximum value.

- 14.1 The time constant of the LR circuit has two interpretations similar to that of the CR circuit.
- a. The current in the circuit will rise by 63% of its maximum value in one time constant.
 - b. The current will rise to its maximum value in (L/R) seconds if the initial rate of increase is maintained.

Note These statements relate to the current flowing in a LR circuit. The equivalent statement for the CR circuit relate to the voltage on the capacitor.

15.

LONG, MEDIUM AND LONG TIME CONSTANT LR CIRCUITS

The definitions for long, medium and short time constants also apply to LR circuits.

i.e.	short time constant circuit	$\frac{L}{R} < \frac{t}{10}$
	long time constant	$\frac{L}{R} > 10t$
	medium time constant	$10t > \frac{L}{R} > \frac{t}{10}$

where t is the pulse duration of the input waveform.

15.1 Practical

Build the circuit of figure 29. Apply the waveform indicated from the signal generator and display the input and output waveforms on an oscilloscope.

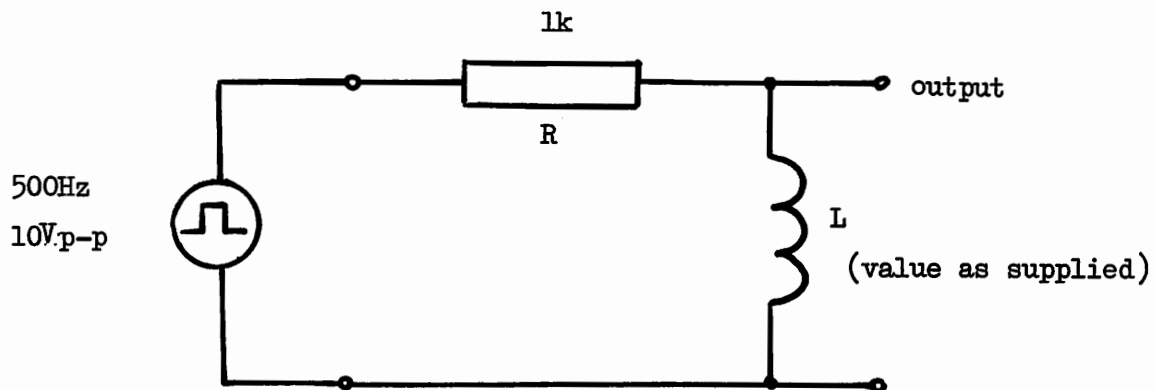


Figure 29 Series LR Circuit

- a. By considering the output waveform what sort of time constant would you consider the circuit to be

LONG / MEDIUM / SHORT

(delete the incorrect statement)

- b. Would you be able to obtain approximate differentiation or integration using this circuit?

- c. To obtain the differential of the input waveform in a CR circuit the output is taken across the:

CAPACITOR / RESISTOR

(delete the incorrect statement)

- d. To obtain the differential of the input waveform in an LR circuit the output is taken across the:

INDUCTOR / RESISTOR

(delete the incorrect statement)

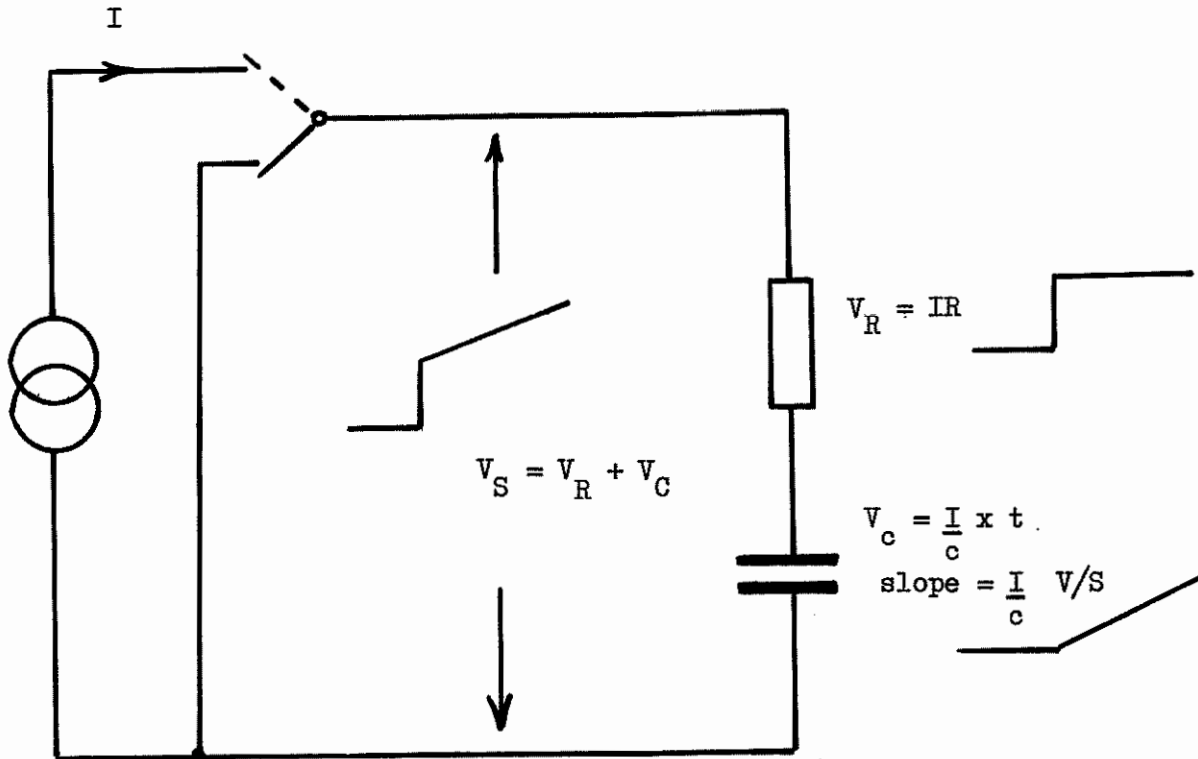
Answers to paragraph 15.1

- a. short
- b. differentiation
- c. resistor
- d. inductor

16.

SUMMARY

16.1 Current step applied to a series CR circuit

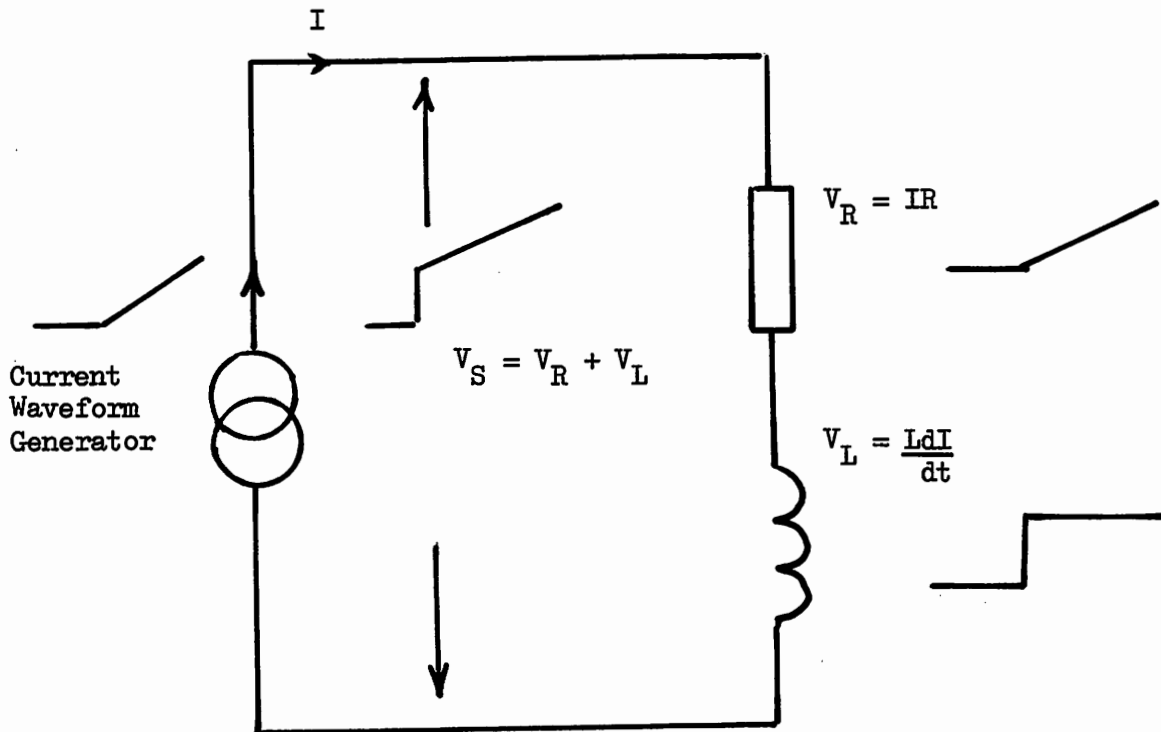


The effect of passing a CONSTANT current through a series CR is illustrated in the above diagram

- a. A constant voltage appears across the resistor
- b. A voltage ramp appears across the capacitor
- c. The voltage across the circuit is the sum of these two waveforms.

16.2 A Triangular current waveform applied to a series LR circuit

The effect of passing a RAMP or SAWTOOTH current waveform through a series LR circuit is illustrated in the diagram below :



- A constant voltage appears across the inductor
- A voltage ramp appears across the resistor
- The voltage across the circuit is the sum of these two waveforms.

NOTE

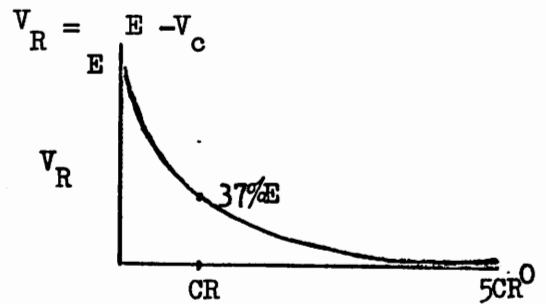
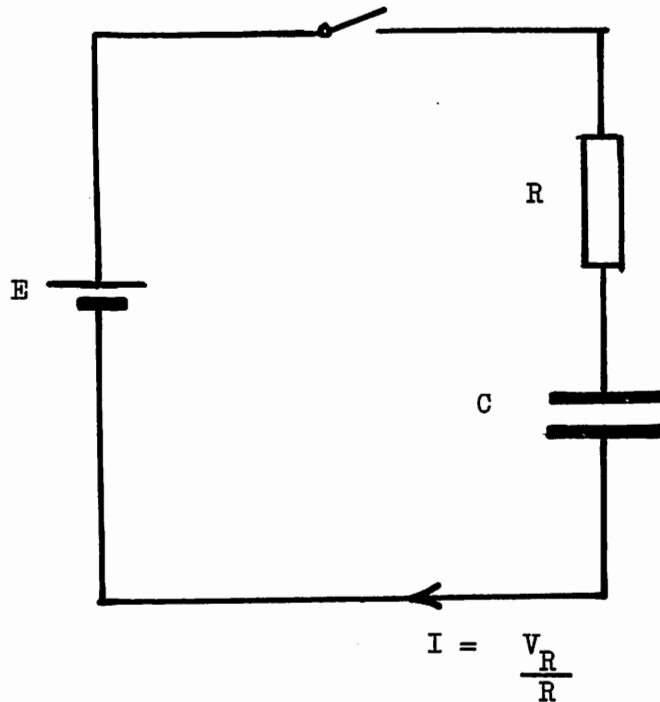
The current waveforms considered so far,

- a constant current through a CR circuit
- a ramp current waveform through an LR circuit

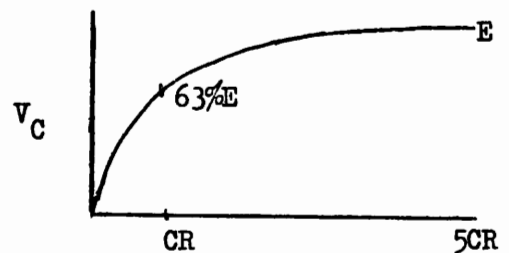
produce constant or ramp shaped waveforms NOT EXPONENTIAL voltage waveforms, these are associated with constant voltage supplies in the next section.

16.3 A Voltage step applied to a series CR circuit

When switch closed



$$V_C = E(1 - \exp \frac{-t}{CR})$$



Time constant = CR seconds

$V_C = 63\% E$ after one time constant

$V_C \approx E$ after five time constants

during discharge $V_C = V \exp \frac{-t}{CR}$

16.4 CR circuits and repetitive voltage waveforms

SHORT CR

Time constant $< \frac{t}{10}$

LONG CR

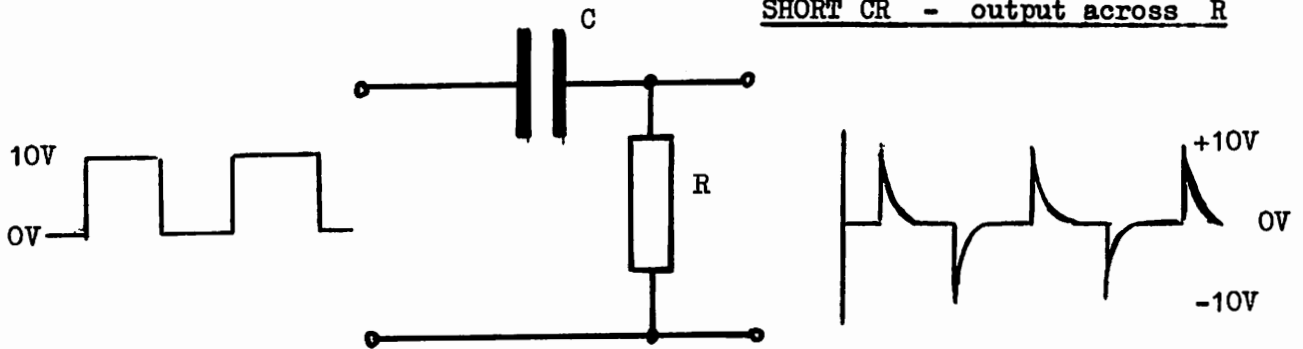
Time constant $> 10t$

MEDIUM CR

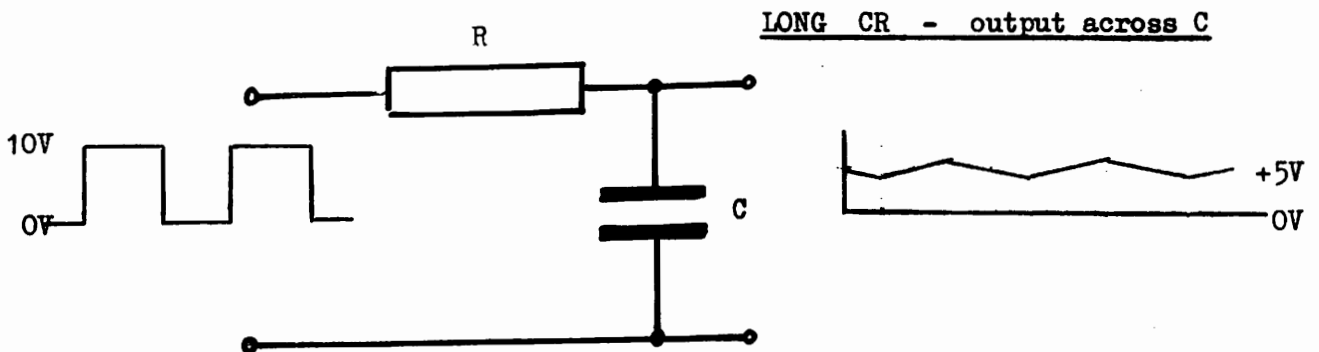
Time constant $> \frac{t}{10}$ but $< 10t$

where t is the pulse duration of the input waveform.

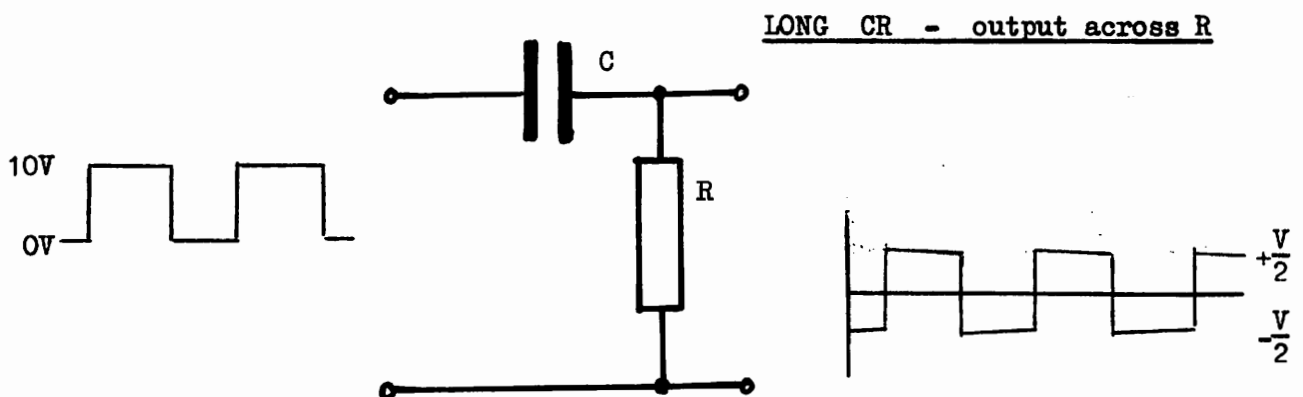
16.5 Differentiating Circuit



16.6 Integrating Circuit

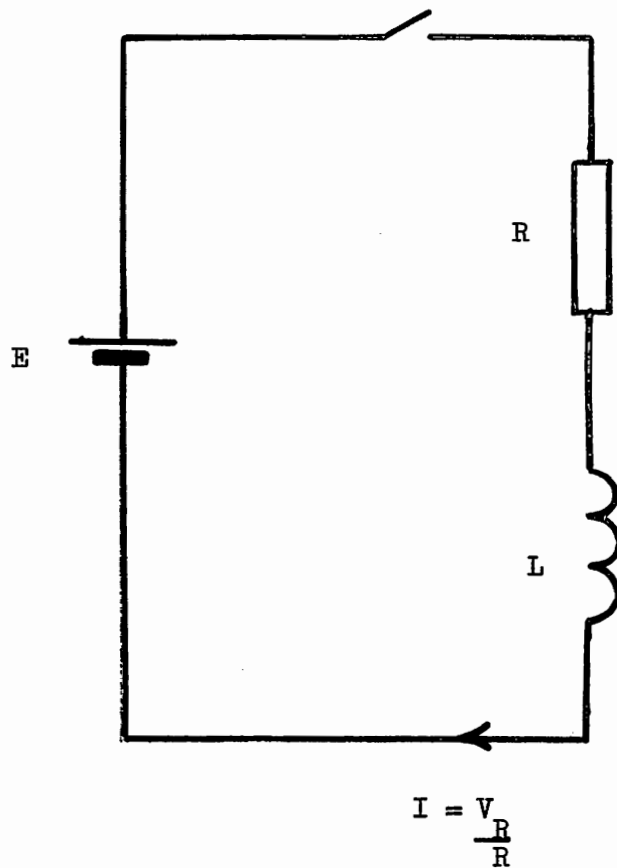


16.7 Coupling Circuit or DC Block



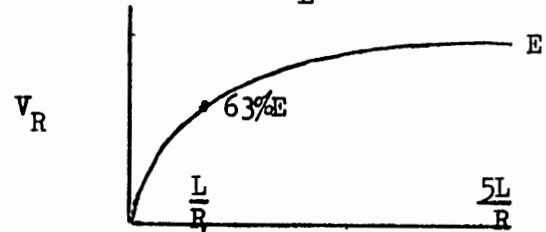
NOTE - The time constant of the circuit is related to the pulse duration (t) of the input waveform, NOT its periodic time (T).

16.8 A Voltage Step Applied to a Series LR Circuit

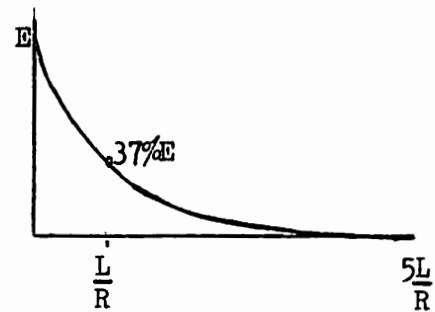


when switch closed

$$V_R = E(1 - \exp \frac{-tR}{L})$$



$$V_L = E - V_R$$



Time constant = $\frac{L}{R}$ seconds

$I = 63\% I_{\max}$ after one time constant

$I \approx I_{\max} = \frac{E}{R}$ after five time constants.

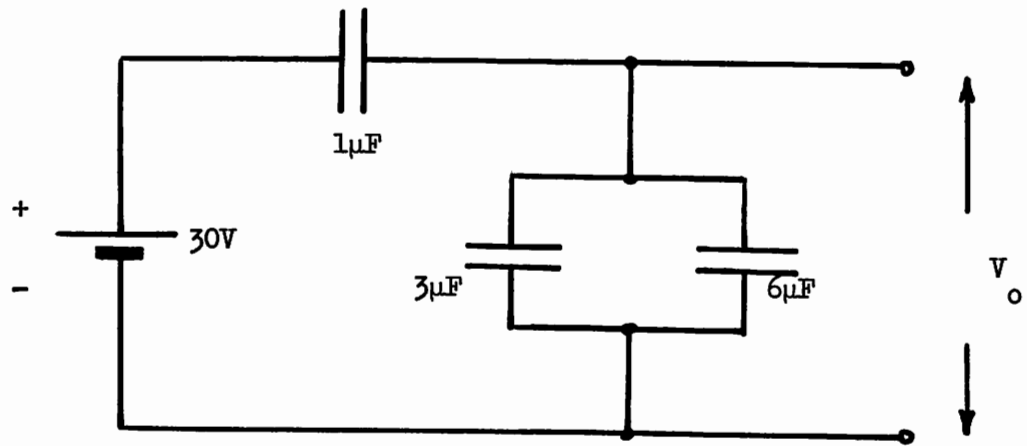
16.9 Short, Medium and Long time constant circuits

Similar to CR circuits.

FURTHER EXAMPLES

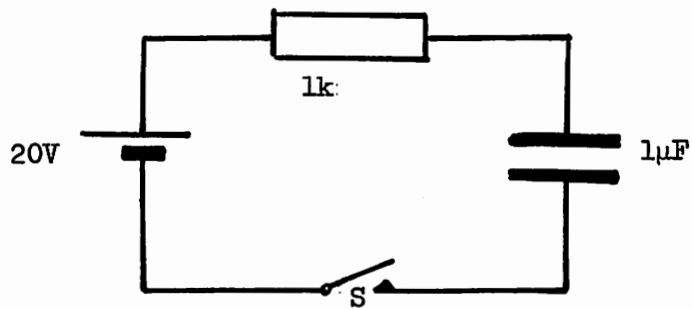
17.

17.1



Calculate the value of V_o .

17.2

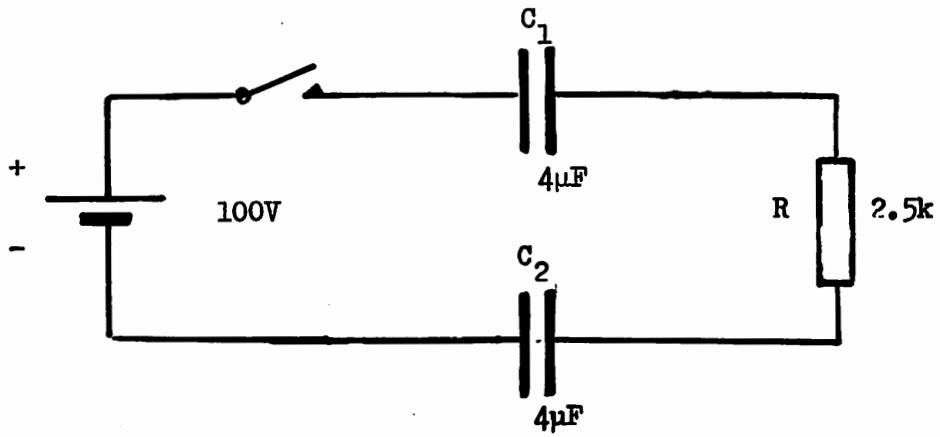


If the capacitor is initially discharged draw a graph of capacitor voltage and resistor voltage against time for the time from closing switch S to when a steady-state condition has been reached.

What would the values of V_C and V_R be after

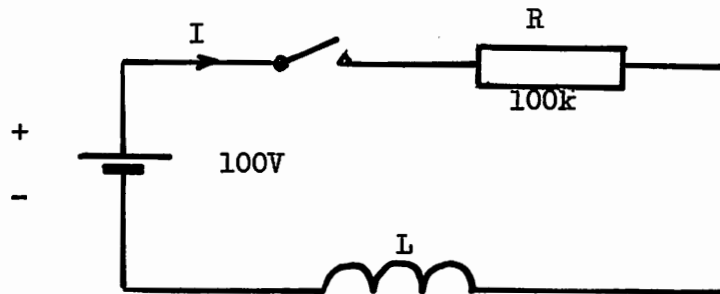
- i. 1 ms
- ii. 2.5 ms
- iii. 5 ms

17.3



Calculate the charge on C_2 5ms after the switch is closed.

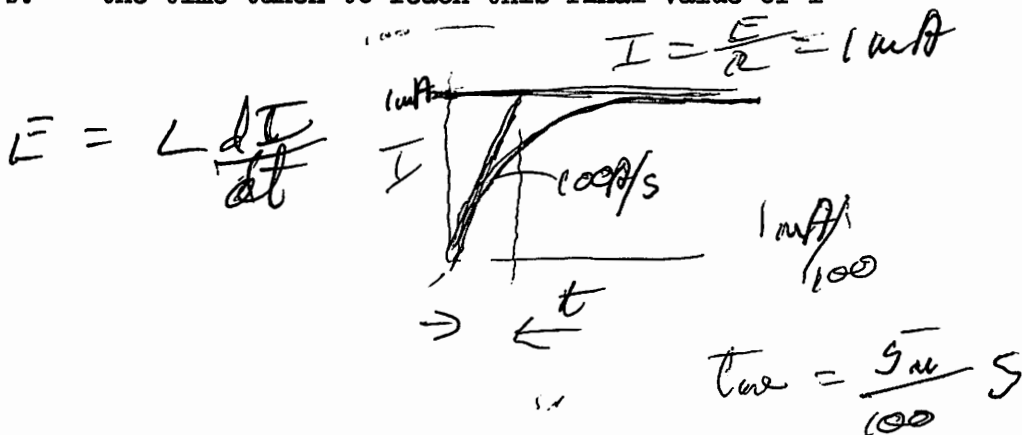
17.4



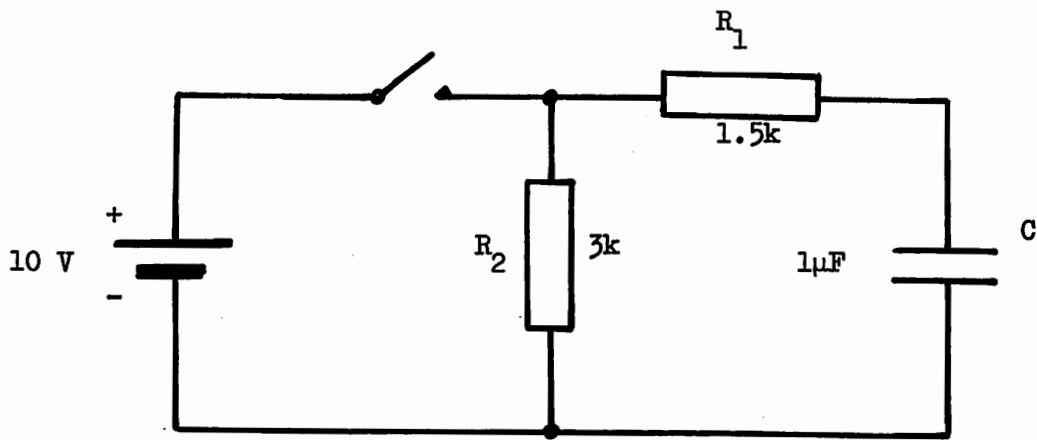
The initial rate of change of the current in the above circuit when the switch is closed is 100A/s .

Calculate:

- the final value of I
- the time taken to reach this final value of I

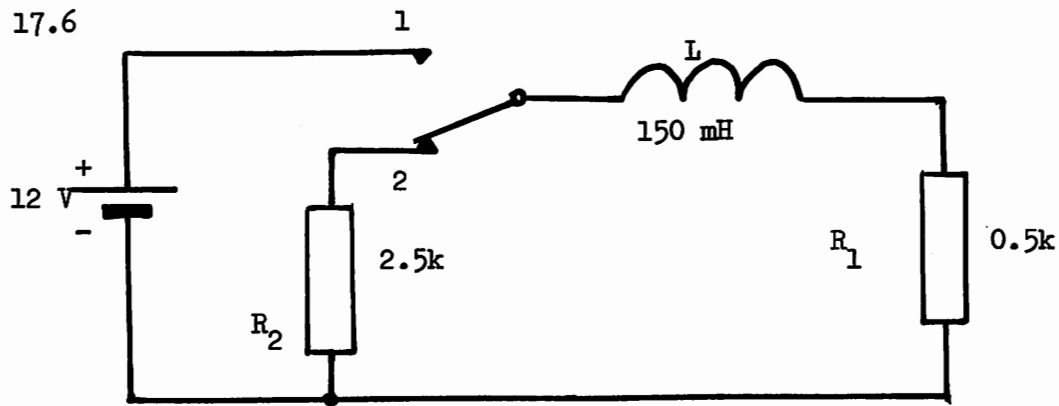


17.5



Initially the switch is open and the capacitor is discharged. The switch is closed for 3 ms and then opened again.

Sketch graphs of V_C and I (the current through the capacitor) against time.



Initially the switch is in position "2", and no current is flowing in the inductor. The switch is then moved to position "1" for $150\mu\text{s}$ and then back to position "2".

Sketch graphs of V_L and I (the current in the inductor) against time. What is the maximum voltage which appears across R_1 ? (Assume the switch contacts change instantaneously).

17.7 A relay has an inductance of 500 mH and a resistance of 1k. It is connected to a 12 Volt supply.

Calculate the time required to operate the relay if it pulls in at 7mA.

- 17.8 a. Sketch a circuit that will integrate a rectangular waveform.
- b. If the resistor value is 10k determine a suitable value for the capacitor. The input has a frequency of 500Hz.

- 17.9 a. Sketch a diagram showing a circuit that will differentiate a 500Hz squarewave.
- i. using a capacitor
 - ii. using an inductor
- b. Determine suitable values for the capacitor and inductor if the resistor value is 1k.

18. ANSWERS

17.1 3V

17.2 V_C - a rising exponential

V_R - a falling exponential

1ms $V_C = 12.64V$ $V_R = 7.36V$

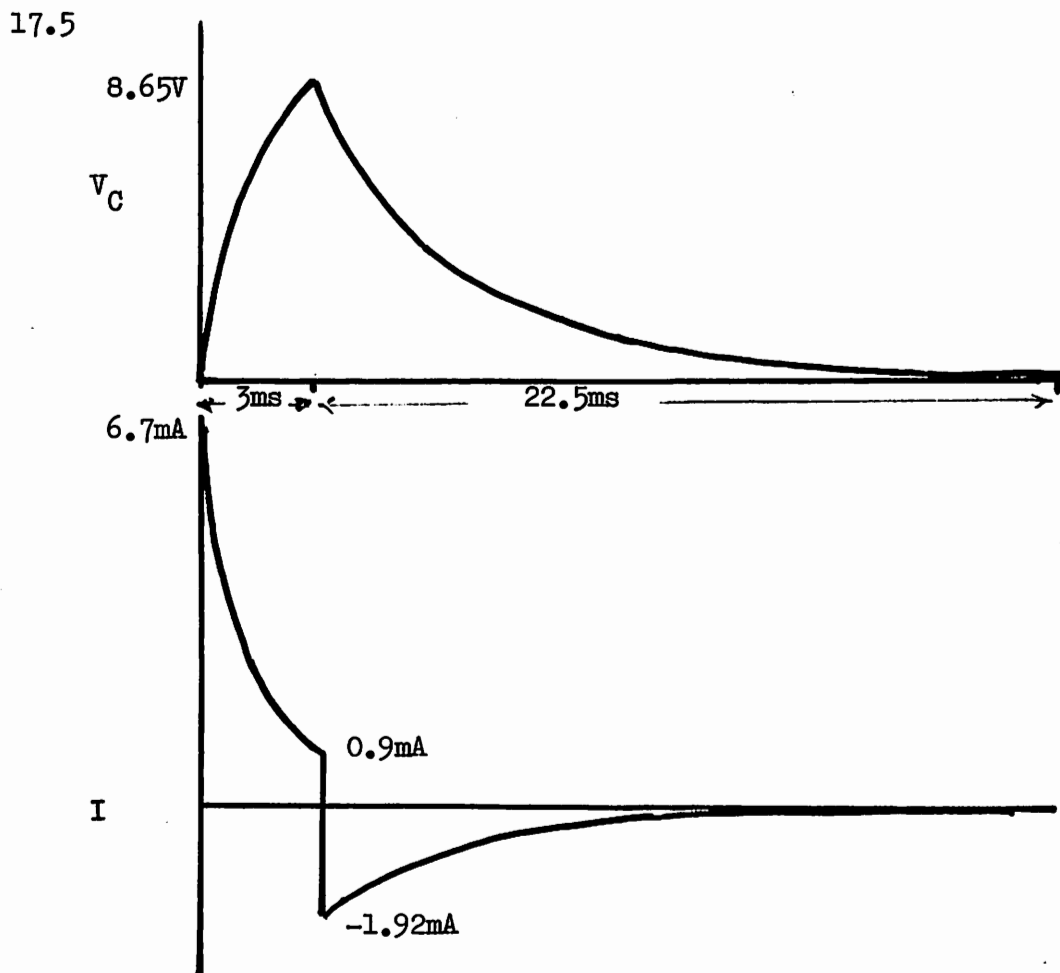
2.5ms $V_C = 18.36V$ $V_R = 1.64V$

5ms $V_C \approx 20V$ $V_R \approx 0V$

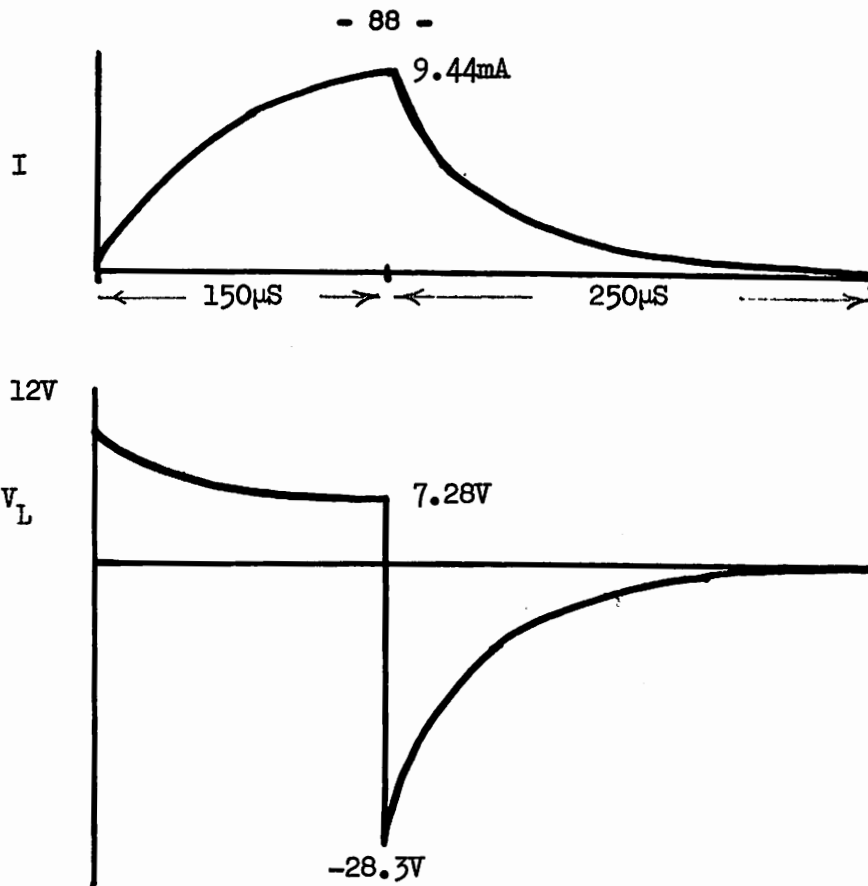
17.3 $Q = 126\mu C$

17.4 Final value of $I = 1mA$

Time taken to reach 1mA = 50 μs



17.6



Maximum voltage across R_1 occurs $150\mu\text{s}$ after the switch is moved to position 1 when $V_{R1} = 9.44 \times 0.5 = 4.72 \text{ V}$

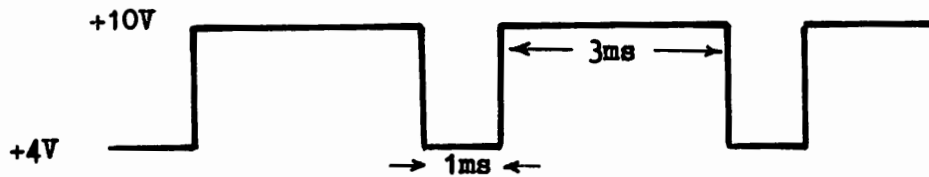
17.7 Time taken = $438\mu\text{s}$.

17.8 Series CR circuit with output across C
C should be not less than $1\mu\text{F}$

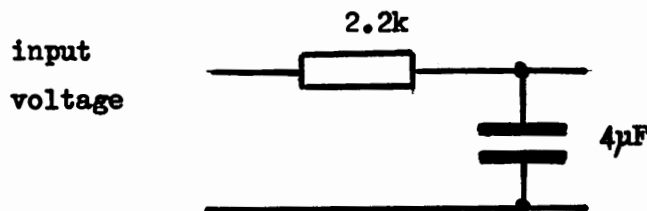
17.9 (i) Series CR circuit output across R. $C \leq 100\text{nF}$
(ii) Series LR circuit output across L. $L \leq 100\text{mH}$

DC TRANSIENTS - FURTHER EXAMPLESB PART 1

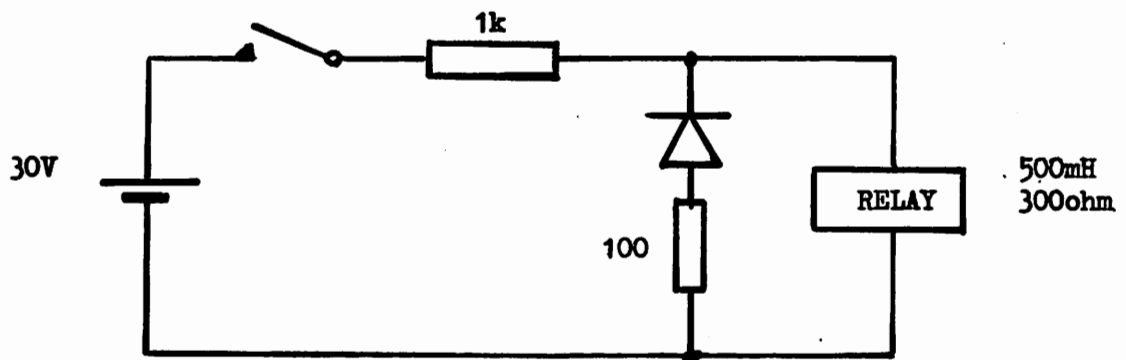
1. A voltage or current source can be applied to any of the following circuits ,
 - (i) a resistance of 100 ohm ,
 - (ii) an inductance of 5H,
 - (iii) a capacitance of 100 μ F,
 - (iv) a coil which has an inductance of 5H and a resistance of 100 ohm ,
 - (v) a 100 μ F capacitor in series with a 100 ohm resistor.
- a. Sketch the voltage waveform developed across each circuit over a period of 30ms when each circuit is connected in turn to a constant current source of 300mA .
- b. Repeat part (a) if the current source now rises linearly from 300mA to 1A over a period of 100ms .
- c. Sketch the current waveform through each circuit if a voltage source of 60V is applied to each of the circuits. Sketch the waveform over a period of time so that, where relevant, the time the current reaches maximum or minimum can be shown.



2. The waveform drawn above is to be differentiated using a CR circuit in which $R = 10k$.
 - a. Draw a suitable circuit diagram.
 - b. Determine a suitable value for the capacitor explaining the reason for choosing such a value.
 - c. Sketch carefully the output waveform from the circuit, indicating on it significant times and voltage levels.
3. The waveform at the top of the page is to be integrated using a CR circuit in which $C = 10 \mu F$. Repeat parts a, b and c as in question 2.
4. An amplifier interstage coupling circuit comprises a $0.22 \mu F$ capacitor and a $4.7k$ resistor. A $3kHz$ symmetrical squarewave varying between +3V and +10V is applied to the circuit.
 - a. Sketch a suitable circuit diagram.
 - b. Sketch the output waveform showing the important voltage levels.
 - c. Calculate the 'sag' produced by the coupling circuit as a fraction of the waveform's peak to peak amplitude expressed as a percentage.



5. a. The capacitor in the circuit drawn above is initially charged to +10V when the input voltage is increased to +25V. How long will it take the capacitor to charge to +20V?
- b. When the voltage on the capacitor reaches +20V the input voltage is changed so as to discharge the capacitor so that it reaches zero volts in 5ms. Determine the value of the voltage applied.

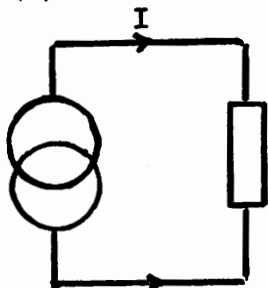


6. The circuit shown above is for operating a 300 ohm relay.
- When will the diode in the circuit conduct ?
 - Calculate the maximum current in the diode.
 - Calculate the pd across the switch contacts when the switch is opened .
 - The relay contacts fall out when the current in the relay coil falls below 10mA. Determine the delay in the relay de-energising when the switch is opened .

ANSWERS TO FURTHER EXAMPLES

1. Circuit

(i)

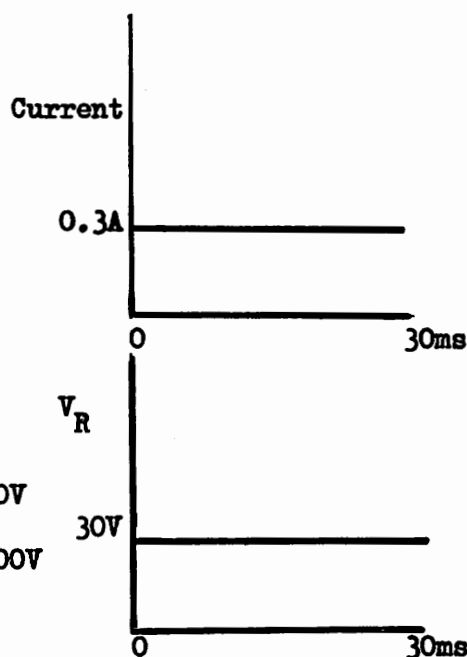


$$V_R = IR$$

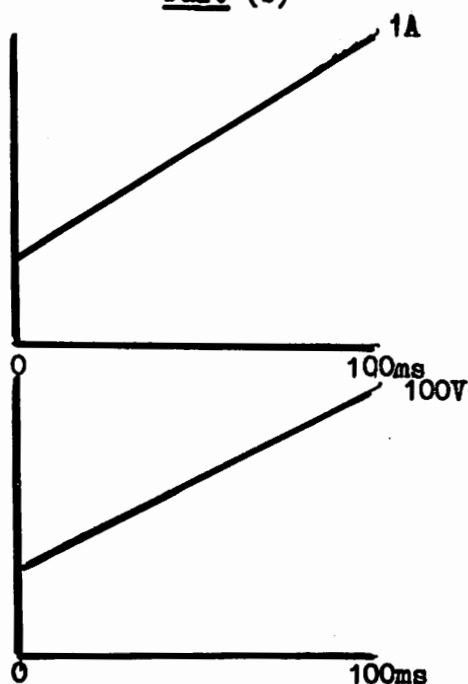
when $I = 0.3A$ $V_R = 30V$

when $I = 1A$ $V_R = 100V$

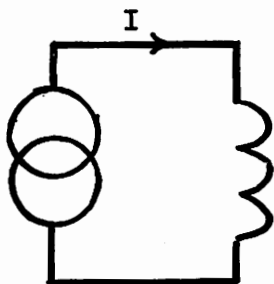
Part (a)



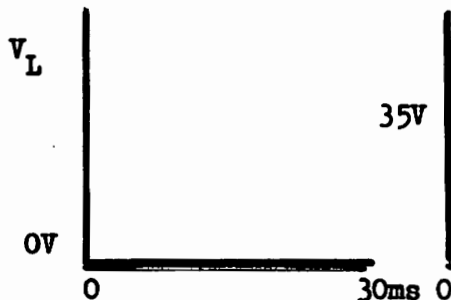
Part (b)



(ii)



$$V_L = L \frac{dI}{dt}$$

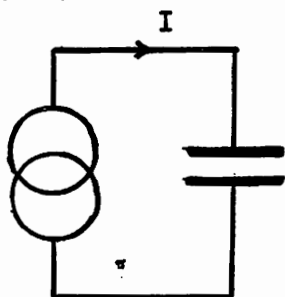


I constant hence

$$V_L = 0$$

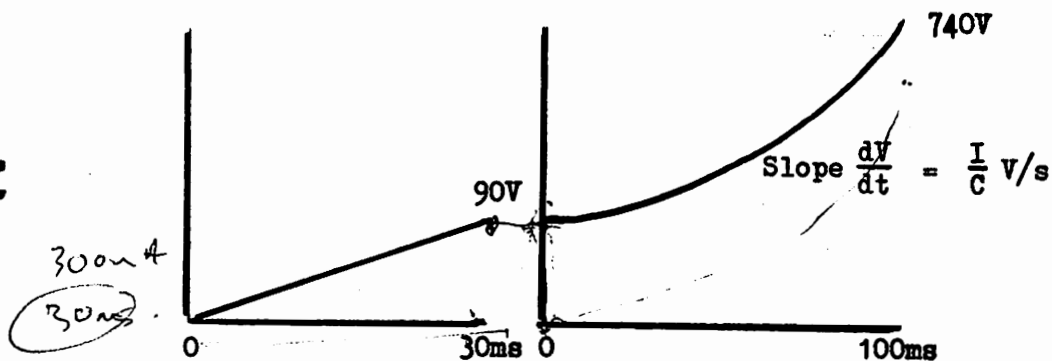
$$V_L = 5H \times \frac{(1A - 0.3A)}{100ms} = 35V$$

(iii)



$$V_C = \frac{Q}{C}$$

$$= \frac{I \times t}{C}$$



I constant at 0.3A hence

$$V_C \text{ will rise linearly to } \frac{0.3A \times 30ms}{100\mu F} = 90V$$

see next page
for part (b)

1.(iii) part(b)

The current charging the capacitor rises from 0.3A to 1A

the increase in voltage at any instant being given by

$$\frac{dV}{dt} = \frac{I}{C} \text{ volts/s .Over the 100ms the average current}$$

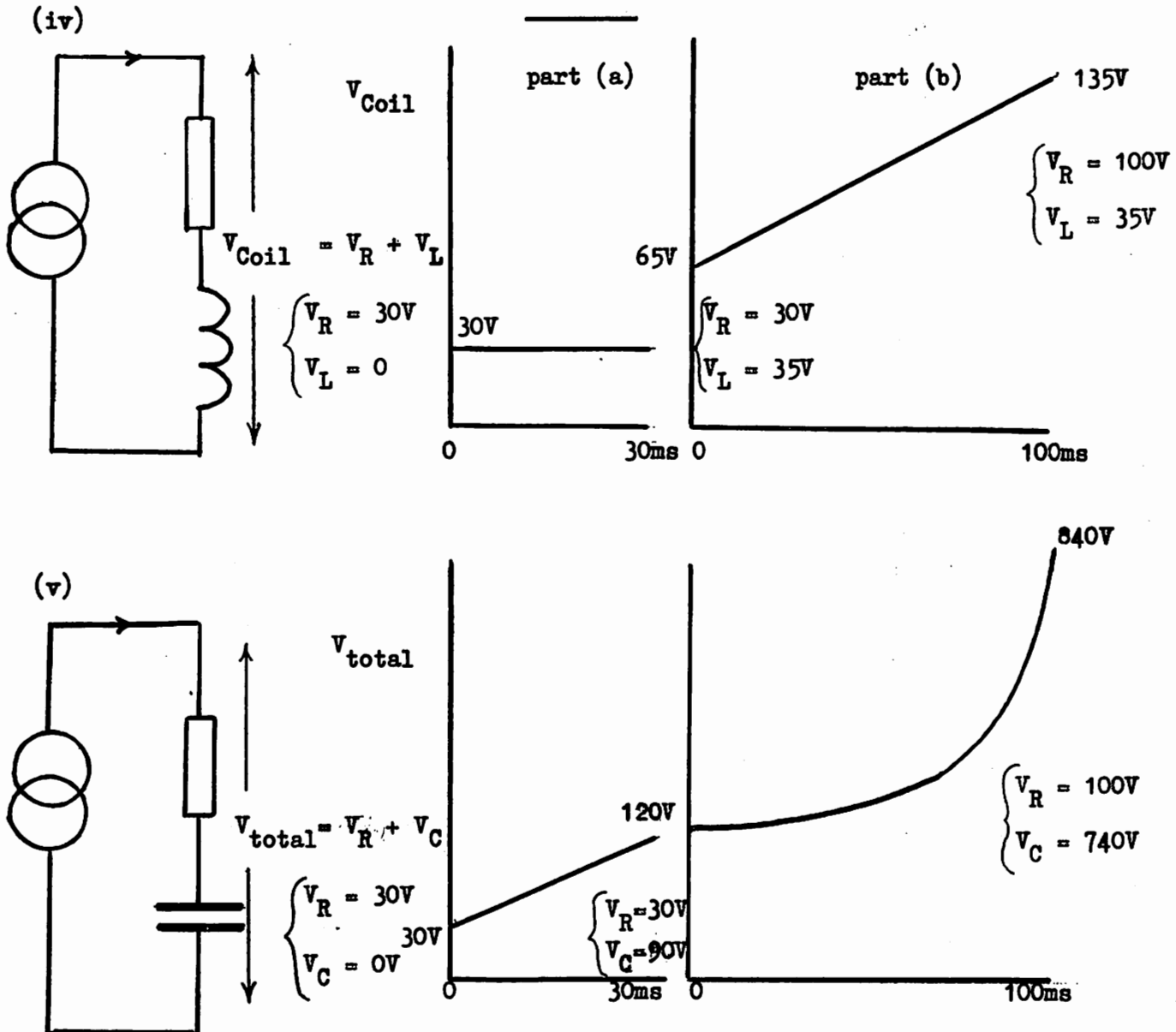
$$\text{will be } \frac{1}{2}(I_{\max} + I_{\min}) = \frac{1}{2}(1A + 0.3A) = 0.65A$$

$$\text{The rise in the voltage on the capacitor} = \frac{I_{\text{av}} \times t}{C}$$

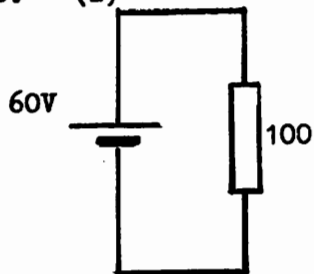
$$= \frac{0.65A \times 100ms}{100\mu F} = 650V$$

Therefore final voltage on the capacitor

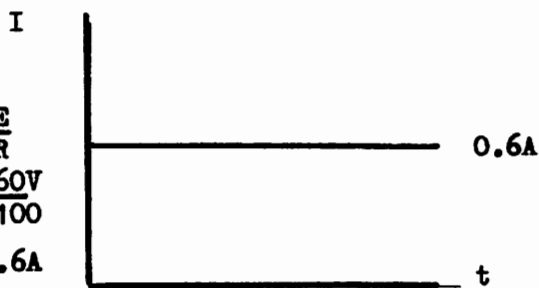
$$90V + 650V = 740V$$



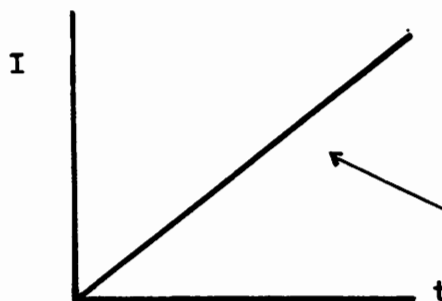
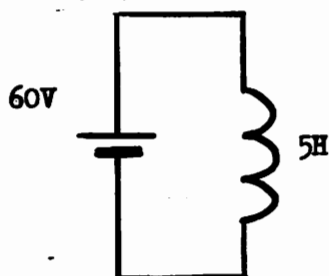
1.c. (i)



$$I = \frac{E}{R} = \frac{60V}{100} = 0.6A$$



(ii)

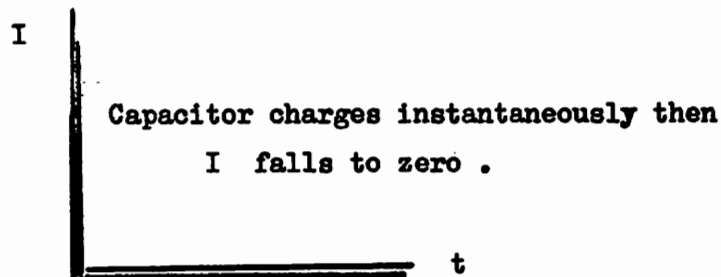
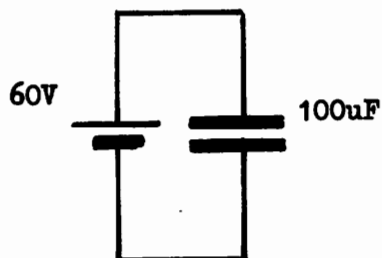


$$E = L \frac{dI}{dt}$$

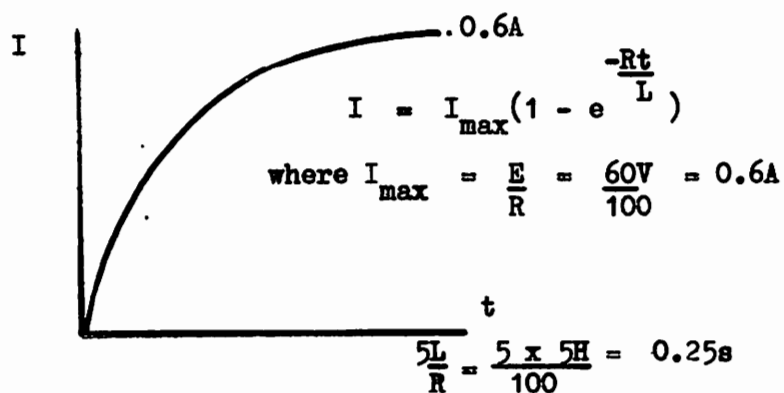
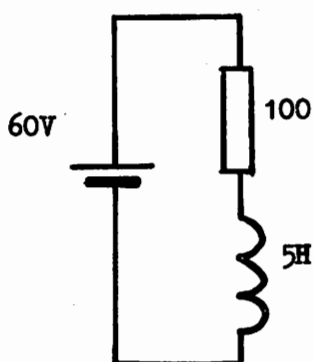
$$\frac{dI}{dt} = \frac{E}{L}$$

Slope = $\frac{60V}{5H} = 12 \text{ A/s}$

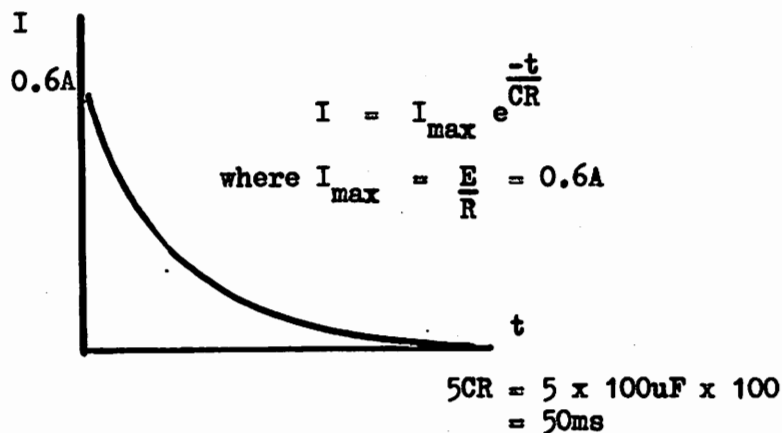
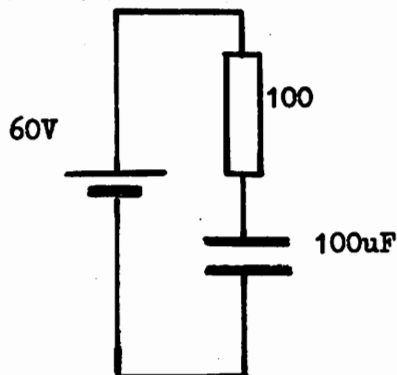
(iii)



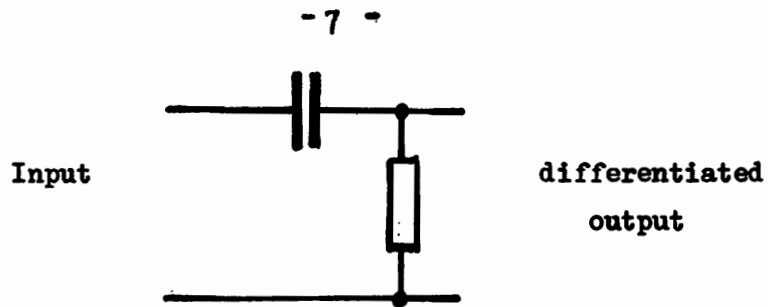
(iv)



(v)



2. a.



b. For the circuit to act as a differentiating circuit it must be a short CR .

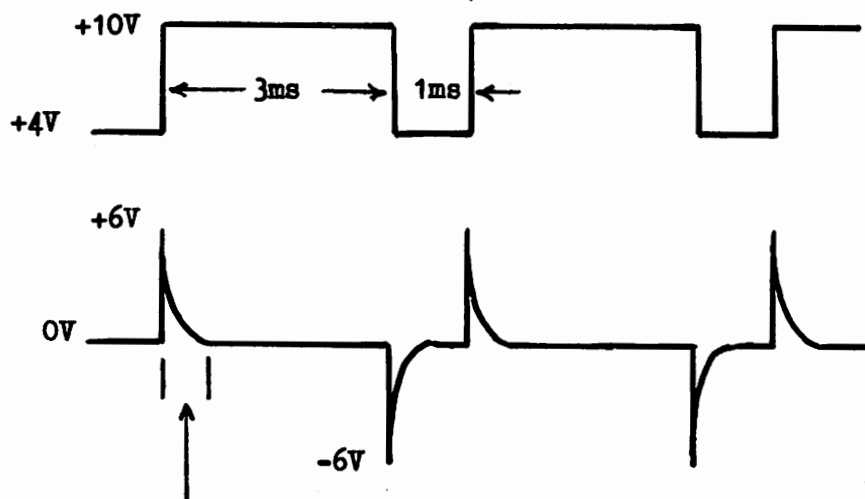
$$\text{ie } CR \ll \frac{t}{10}$$

If we make $t = 3\text{ms}$ then the capacitor will not be able discharge within the negative going part of the pulse, hence we need to make $t = 1\text{ms}$, the narrower of the two pulse durations.

$$C \leq \frac{1\text{ms}}{10 R}$$

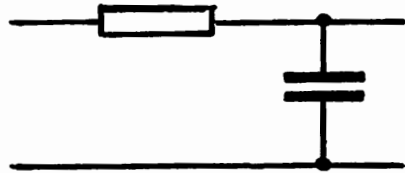
$$\underline{C \leq 10\text{nF}}$$

c.



Capacitor will charge in $5CR$ seconds
 $= 0.5\text{ms}$

3. a.



Integrating circuit

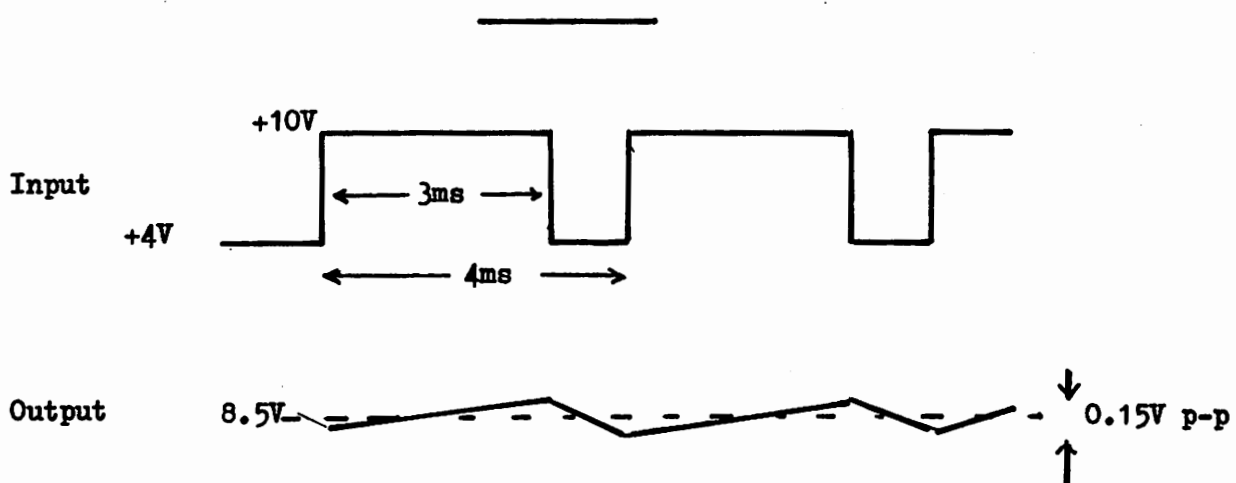
b. For the circuit to act as an integrating circuit it must be a long CR to both half cycles of the input waveform

$$\text{ie} \quad CR \gg 10t$$

t should be 3ms for this relationship to be true for both half cycles .

$$R \gg \frac{10 \times 3\text{ms}}{10\mu\text{F}}$$

$$\gg 3\text{k}$$



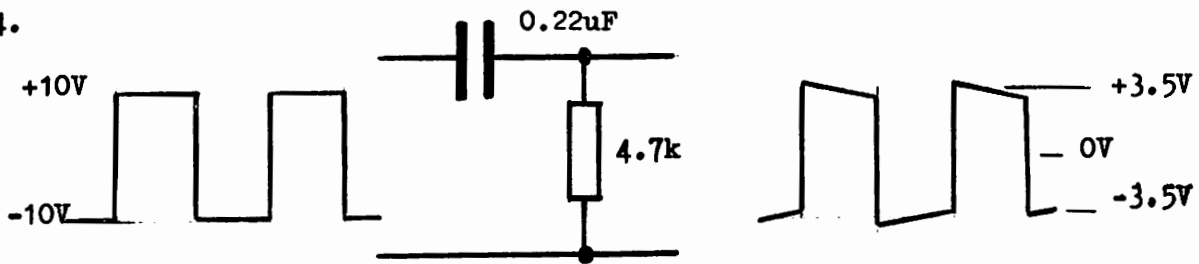
$$\begin{aligned} \text{The mean voltage on the capacitor} &= 4V + \left(\frac{3}{4} \times 6\right)V \\ (\text{mean level of input}) &= 8.5V \end{aligned}$$

During the 3ms pulse the capacitor will charge from a voltage just below +8.5V aiming for the input voltage of +10V . Assuming the capacitor charges linearly then the capacitor voltage will rise by $\frac{I}{C} \times t$ where $I = \frac{V_R}{R}$

$$\Delta V_C = \frac{1.5V \times 3\text{ms}}{10\mu\text{F} \times 3\text{k}}$$

$$= 0.15V$$

4.



$$\text{Pulse duration of waveform} = \frac{1}{2f} = \frac{1}{2 \times 3\text{kHz}} = 0.167\text{ms}$$

$$\text{Circuit time constant} = CR = 0.22\mu\text{F} \times 4.7\text{k} = 1.03\text{ms}$$

the circuit is therefore approaching what can be defined as a long time constant.

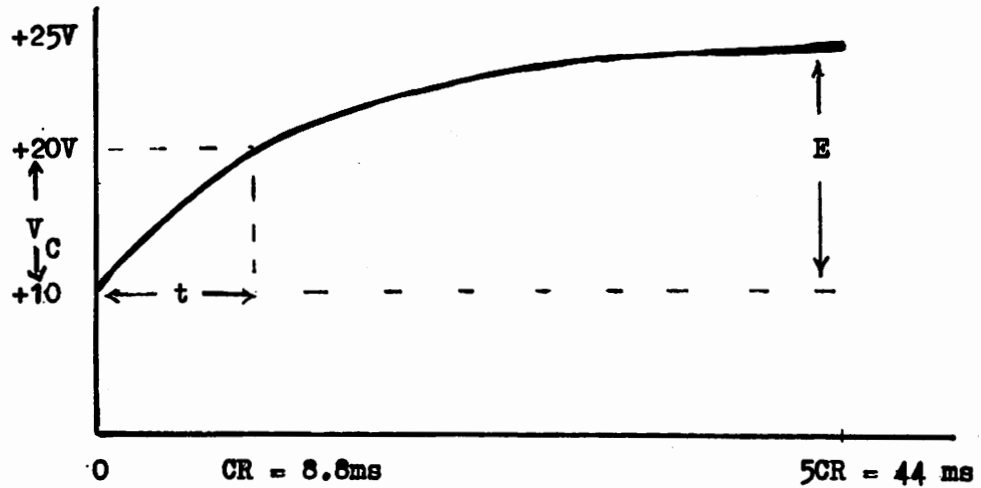
If the circuit was a very long time constant then the mean level of the output waveform would be 0V, switching between +3.5V and -3.5V with negligible sag. However because the time constant is not infinitely large the capacitor will charge and discharge. On the positive going half cycle of the waveform the voltage across the resistor will start just above +3.5V and will fall as the capacitor charges. The fall in the voltage being given by

$$\begin{aligned} V &= \frac{I}{C} \times t = \frac{V_R \times t}{R \times C} \\ &= \frac{3.5\text{V} \times 0.167\text{ms}}{4.7\text{k} \times 0.22\mu\text{F}} \\ &= 0.56\text{V} \end{aligned}$$

Hence sag on the output will be 0.56V

$$\text{and percentage sag} = \frac{0.56\text{V}}{7\text{V}} \times 100 = 8\%$$

5.a



If we consider the general formula for the charge of a capacitor

$$V_C = E \left(1 - e^{-\frac{t}{CR}} \right)$$

we can see from the sketch of the voltage on the capacitor that in this case

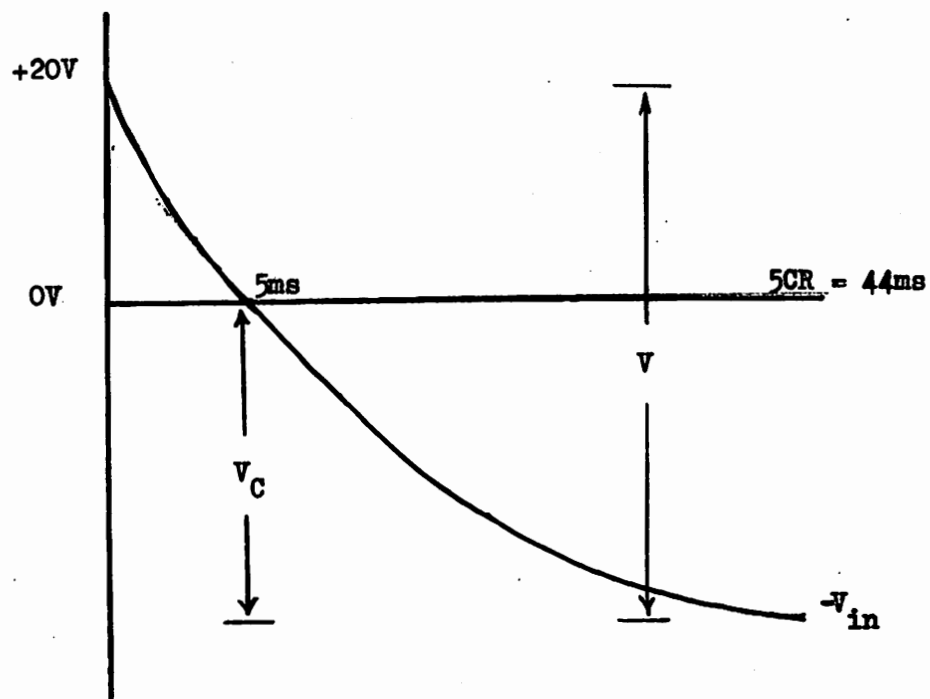
$$V_C = 20V - 10V = 10V$$

$$E = 25V - 10V = 15V$$

hence

$$\begin{aligned}
 t &= -CR \ln \left(1 - \frac{V_C}{E} \right) \\
 &= -4\mu F \times 2.2k \times \ln \left(1 - \frac{10}{15} \right) \\
 &= 9.67ms
 \end{aligned}$$

b.



- 5 b. The general equation for the discharge of a capacitor is given by

$$V_C = V e^{-\frac{t}{CR}}$$

This is the equation for a falling exponential, consideration of the sketch of the voltage on the capacitor, drawn on the previous page, shows that in relation to this equation E must equal $(V_{in} + 20V)$ and after a time t seconds V_C will be equal to V_{in}

$$\text{hence } V_{in} = (V_{in} + 20V) e^{-\frac{5ms}{4\mu F \times 2.2k}}$$

$$V_{in} = \frac{20 e^{-0.568}}{1 - e^{-0.568}} = 26.1V$$

$$\text{Applied voltage} = -26.1V$$

- 6.a. When the switch is opened the relay inductance will generate an emf of such a polarity and magnitude to keep the current flowing hence the back-emf will cause the diode to conduct.

b. Maximum current in the relay = $\frac{E}{R_{total}} = \frac{30V}{1.3k} = 23.1mA$

If the switch is now opened this current will now flow through the diode.

- c. When the switch is opened the pd across the 100ohm resistor will be

$$V_R = I \times R = 23.1mA \times 100 = 2.31V$$

hence the pd across the switch contacts will be

$$V = 30V - (-2.31V) = 32.31V$$

- d. Current decay is given by $I = I_{max} e^{-\frac{Rt}{L}}$

$$\text{Where } I = 10mA \text{ and } I_{max} = 23.1mA$$

$$t = -\frac{L}{R} \ln \frac{I}{I_{max}} = -\frac{0.5}{400} \ln \frac{10mA}{23.1mA} = 1.05ms$$

P.A. Lacy

Nov. 83